

206171 REVIEW EXERCISE (FINAL 1/2019)

1. Find each limit. If the limit is infinity, write either $+\infty$ or $-\infty$.

a) $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x^2 + 2x + 5}}{x + 1}$

d) $\lim_{x \rightarrow 0.5^+} \frac{\ln x}{x - 0.5}$

b) $\lim_{x \rightarrow -1^+} \frac{x^2 - 1}{|x - 1| - 2}$

e) $\lim_{x \rightarrow +\infty} \frac{x^{-2} + x^{-3} + 1}{e^{-x} + 2}$

c) $\lim_{x \rightarrow +\infty} \frac{3x^{10} + 7x^2 - x + 1}{-2x^8 + 4}$

f) $\lim_{x \rightarrow -\infty} (-2x^3 + 3x^2 + 1)$

2. Show how to find the following limits.

a) $\lim_{x \rightarrow -\infty} \sqrt{\frac{4x^6 + 8x^{10}}{4x^{10} + 5x^2}}$

b) $\lim_{x \rightarrow 1} \frac{|2x + 1|}{x^2 - 1}$

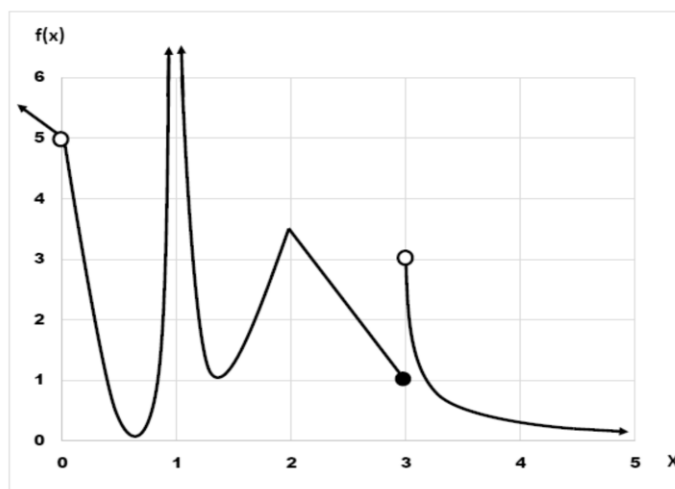
3. Define $f(x) = \begin{cases} ax^2 + 1, & x \geq 1 \\ x + 3, & x < 1. \end{cases}$ If $\lim_{x \rightarrow 1} f(x)$ exists, find a .

4. Show how to find the following limits.

a) $\lim_{x \rightarrow 1^+} \frac{\ln x}{e^x - ex}$

b) $\lim_{x \rightarrow -\infty} \frac{e^{-2x}}{x^2 + 1}$

5. Consider the graph of function f below and answer the following questions.



a) $\lim_{x \rightarrow +\infty} f(x) =$ $\lim_{x \rightarrow 3^-} f(x) =$ $\lim_{x \rightarrow 1^-} f(x) =$ $\lim_{x \rightarrow 3^+} f(x) =$

b) f is not continuous at

c) Are there any values of x where f is continuous but nondifferentiable? If so, list them.

d) Are there any values of x where f is differentiable but not continuous? If so, list them.

6. Let $f(x) = \begin{cases} A & , \quad x \leq 0 \\ x^2 + 6 & , \quad 0 < x < 2 \\ 5x^2 & , \quad 2 \leq x < 3 \\ 15x & , \quad x \geq 3. \end{cases}$

a) Find A where f is continuous at $x = 0$.

b) Is f continuous at $x = 2$? why?

c) Find $\lim_{h \rightarrow 0^-} \frac{f(3+h) - f(3)}{h}$.

d) Find $\lim_{h \rightarrow 0^+} \frac{f(3+h) - f(3)}{h}$.

e) Is f differentiable at $x = 3$? why?

7. Find the equation of the tangent line to the graph of $f(x) = x^{172} - 170x + 174$ at $x = 1$.

8. Let $y = f(x) = x^{\frac{1}{3}}$.

a) Find the differential dy

b) Use the differential to approximate $(1001)^{\frac{1}{3}}$.

9. The total **cost** of making x pens (in dollars) in a week are given by $C(x) = x + 350$ where $0 \leq x \leq 1,000$. The company fixes **the price for each pen to be 1.7 dollars**. Answer the following questions.

a) Find the revenue function.

b) Find the marginal profit function.

c) What is the marginal profit at the level of 100 pens?

d) What is the exact profit of selling the 101st pen?

10. The total **cost** and the total **revenue** (in dollars) of printing and selling x books are given by functions $C(x)$ and $R(x)$ respectively. The company knows that $C'(106) = 30$, $C'(600) = 15$, $R'(600) = 37$, $R'(601) = 37.5$ and the total **profit** of selling 600 books is 15,000 dollars.

a) Find the marginal profit at $x = 600$. Then interpret the quantity.

b) Estimate the cost of printing the 107th book and the 601st book.

c) Estimate the value of $R(602) - R(600)$.

d) Estimate the profit of selling 605 books.

11. Find the following derivatives.

a) $\frac{d}{dx} \left[\log_2(x) + \sqrt[3]{x} - e^{18} + 2^{8x-1} \right]$

b) $g'(t)$ where $g(t) = (\ln(2-t) + t^{0.6})^{171}$

c) y' where $y = 30^x(x^{11} + 20x - 1)^{-6}$

d) $\frac{df}{dx}$ when $f(x) = \frac{\sqrt{x^2 - 5x}}{5x^9 + 1}$

e) $\left. \frac{dy}{dx} \right|_{x=\pi}$, $y = u^3 + u$, $u = 1 - 4\sin(x)$

12. Given $\frac{dy}{dx} = xy^2 + e^{\cos(y)}$, where y is a function of x , find $\frac{d^2y}{dx^2}$.

13. Find $\frac{dy}{dx}$ of $y = \frac{(1 - \cos 2x)^3 \ln(x)}{(x^2 + 1)^{1/3}}$ using logarithmic differentiation.

14. Given that $y = e^t \sin t$, find $y'' - 2y' + 2y + 1$.

15. Given that $f^{(5)}(x) = 3e^{4x} + (2x+1)^{10}$, find

(a) $f^{(6)}(x)$

(b) $f^{(7)}(0)$

(c) $f^{(100)}(x)$

15. Estimate value of $\sqrt{1.1}$ using the 2nd Taylor polynomials of $f(x) = \sqrt{x}$ about $x = 1$.

16. Let $f(x) = x^{5/3} + 5x^{2/3}$, $f(-2) \approx 4.7$, $f'(x) = \frac{5(x+2)}{3x^{1/3}}$ and $f''(x) = \frac{10(x-1)}{9x^{4/3}}$.

a) Find the domain of f .

b) Find all critical values of f .

c) Find the interval(s) on which f is increasing.

d) Find the interval(s) on which f is decreasing.

e) Find the interval(s) on which f is concave up.

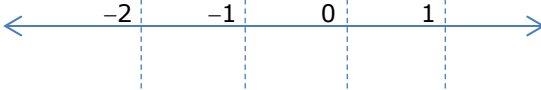
f) Find the interval(s) on which f is concave down.

g) Find all inflection points of f .

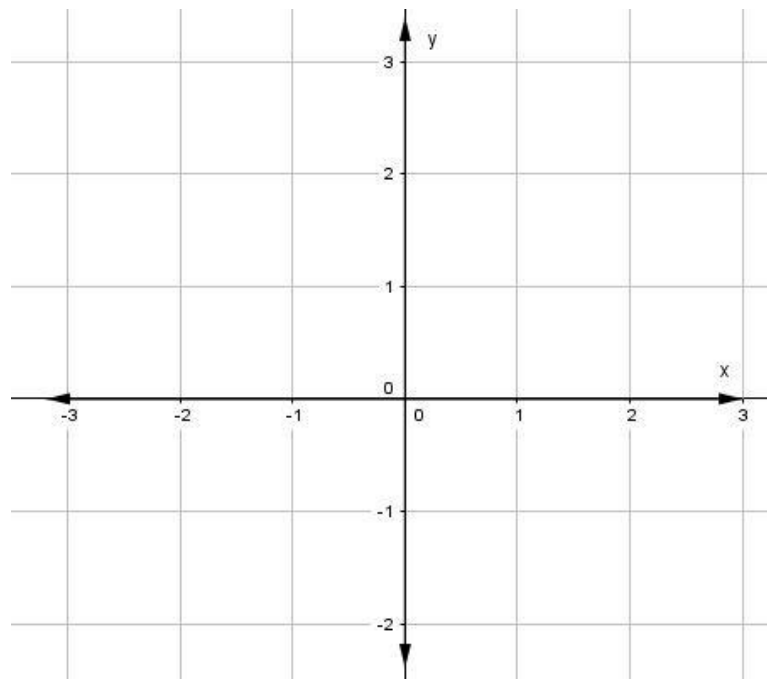
h) Find each local maximum of f .

i) Find each local minimum of f .

17. Let $y = f(x)$ be a continuous function which $D_f = \mathbb{R}$, $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = \infty$. Sketch the graph of f using the below information.

$f(-2) = 0, f(-1) = -1, f(0) = 0, f(1) = 1$
 $f'(x)$ - - - - - 0 + + + ND + + 0 + + +
 $f''(x)$ + + + + + + + + + + 0 - - 0 + + +


where ND = Not Defined, sketch the graph of f .



18. A restaurant makes and sells x cupcakes per day. The daily revenue and cost functions (in baht) are $R(x) = 80x - 0.5x^2$, $C(x) = 250 + 20x + 0.5x^2$, $0 \leq x \leq 40$

What is the maximum daily profit? And how many cupcakes should be made to maximize the daily profit?