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Subject : \_\_\_\_\_

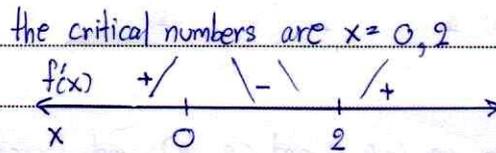
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EXERCISES

1. Let  $f(x) = x^3 - 3x^2 + 1$

(a) Find the critical numbers of  $f$  and the intervals on which  $f$  is increasing, and those on which  $f$  is decreasing.

consider,  $f'(x) = 3x^2 - 6x = 0$   
 $3x(x-2) = 0$   
 $x = 0, 2$



$\therefore f$  is increasing on  $(-\infty, 0) \cup (2, +\infty)$  and decreasing on  $(0, 2)$

(b) Find the local maxima and local minima of  $f$ .

From (a) the critical numbers are  $x = 0, 2$

thus,  $f(0) = 0^3 - 3(0)^2 + 1 = 1$  is the local maximum

$f(2) = 2^3 - 3(2)^2 + 1 = -3$  is the local minimum

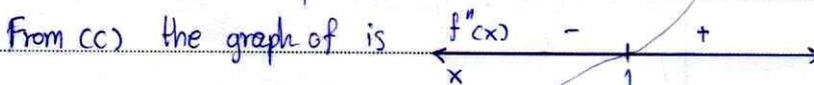
(c) Find the intervals where the graph of  $f$  is concave up or concave down.

consider,  $f''(x) = 6x - 6 = 0$   
 $6(x-1) = 0$   
 $x = 1$



the graph of  $f$  is concave upward on  $(1, +\infty)$  and concave downward on  $(-\infty, 1)$

(d) Find the inflection points of  $f$ .



$f(1) = 1^3 - 3(1)^2 + 1 = -1$

$\therefore (1, f(1)) = (1, -1)$  is an inflection point.

2. Sketch the graph of the following functions  $f$ .

(a)  $f(x) = x^3 - 6x^2 + 9x + 1$

Sol<sup>n</sup> / Steps The domain of  $f$  is  $(-\infty, \infty)$

x intercepts: Solve  $f(x) = 0$

$x^3 - 6x^2 + 9x + 1 = 0$

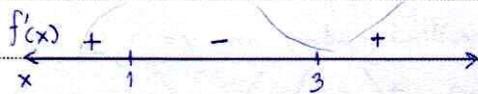
y intercept:  $f(0) = 1$

Step 2.  $f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-3)(x-1)$

Partition numbers for  $f'$ :  $x=1$  and  $x=3$

Critical numbers of  $f$ : 1, 3

Sign chart for  $f'(x)$ :

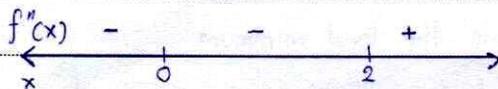


So  $f$  is increasing on  $(-\infty, 1)$  and  $(3, \infty)$  and decreasing on  $(1, 3)$ . Therefore,  $f(1) = 5$  is a local maximum, and  $f(3) = 1$  is a local minimum.

Step 3.  $f''(x) = 6x - 12 = 6(x-2)$

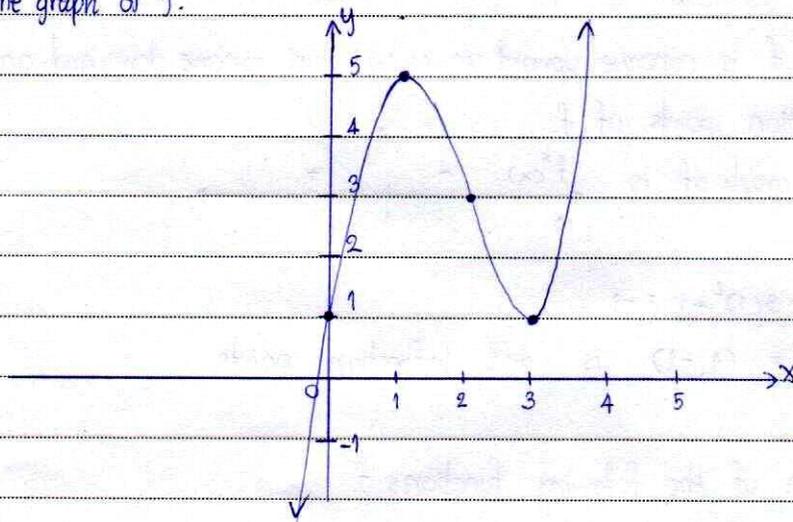
Partition numbers for  $f''$ : 0, 2

Sign chart for  $f''(x)$ :



So  $f$  is concave upward on  $(2, \infty)$ , is concave downward on  $(-\infty, 0)$  and  $(0, 2)$ , and has an inflection point at  $x=2$ . Since  $f(2) = 3$ , the inflection point is  $(2, 3)$ .

Step 4. Sketch the graph of  $f$ .



(b.)  $f(x) = x^3 - 3x$

Sol<sup>n</sup> Step 1. The domain of  $f$  is  $(-\infty, \infty)$

$x$  intercepts: Solve  $f(x) = 0$

$$x^3 - 3x = 0$$

$$x(x^2 - 3) = 0$$

The  $x$  intercept of  $f$  is  $x = 0$

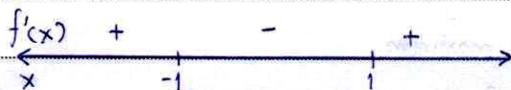
$y$  intercept:  $f(0) = 0$

Step 2  $f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x-1)(x+1)$

Partition numbers for  $f'$ :  $x = -1$  and  $x = 1$

Critical numbers of  $f$ :  $-1, 1$

Sign chart for  $f'(x)$ :

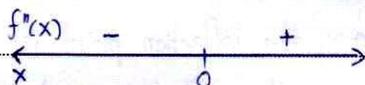


So  $f$  is increasing on  $(-\infty, -1)$  and  $(1, \infty)$  and decreasing on  $(-1, 1)$ . Therefore,  $f(-1) = 4$  is a local maximum, and  $f(1) = -2$  is a local minimum.

Step 3  $f''(x) = 6x$

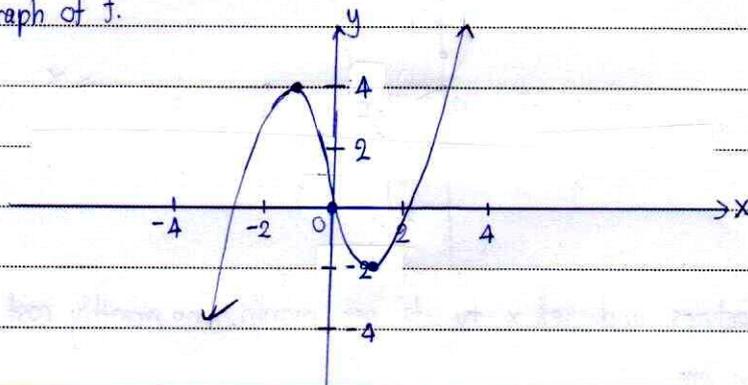
Partition number for  $f''$ :  $0$

Sign chart for  $f''(x)$ :



So  $f$  is concave upward on  $(0, \infty)$ , is concave downward on  $(-\infty, 0)$ , and has an inflection point at  $x = 0$ . Since  $f(0) = 0$ , the inflection point is  $(0, 0)$ .

Step 4. Sketch the graph of  $f$ .



(c)  $f(x) = x^4 - 2x^3$

Sol<sup>n</sup> Step 1. The domain of  $f$  is  $(-\infty, \infty)$

$x$  intercepts: Solve  $f(x) = 0$

$$x^4 - 2x^3 = 0$$

$$x^3(x - 2) = 0$$

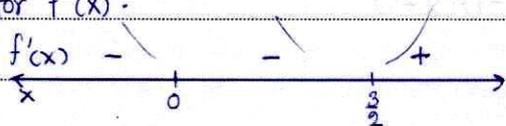
The  $x$  intercepts of  $f$  are  $x = 0$  and  $x = 2$

$y$  intercept:  $f(0) = 0$

Step 2  $f'(x) = 4x^3 - 6x^2 = 2x^2(2x - 3)$

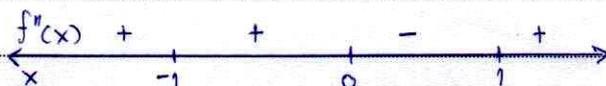
Partition numbers for  $f'$ :  $x = 0$  and  $x = \frac{3}{2}$

Critical number of  $f$ :  $0, \frac{3}{2}$

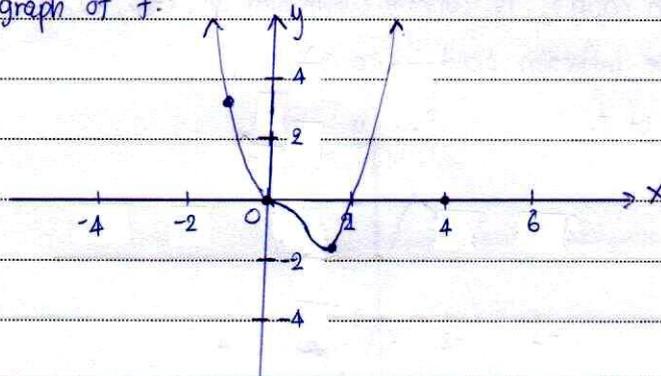
Sign chart for  $f'(x)$ :

So  $f$  is increasing on  $(\frac{3}{2}, \infty)$  and decreasing on  $(-\infty, 0)$  and  $(0, \frac{3}{2})$ . Therefore,  $f(\frac{3}{2}) = 1.7$  is a local minimum and there is no a local maximum.

Step 3  $f''(x) = 12x^2 - 12x = 12x(x^2 - 1) = 12(x-1)(x+1)$

Partition numbers for  $f''(x)$ :  $-1, 0, 1$ Sign chart for  $f''(x)$ :

So  $f$  is concave upward on  $(-\infty, 1)$ ,  $(-1, 0)$ , and  $(1, \infty)$ , is concave downward on  $(0, 1)$ , and has an inflection point at  $x = 0$ , since  $f(0) = 0$ , the inflection point is  $(0, 0)$   
 $x = 1$ , since  $f(1) = -1$ , the inflection point is  $(1, -1)$

Step 4 Sketch the graph of  $f$ .

- (3) A company manufactures and sell  $x$  tv sets per month. The monthly cost and price-demand (in dollars) equation are

$$C(x) = 60,000 + 60x \text{ and } p(x) = 200 - \frac{x}{50} \text{ for } 0 \leq x \leq 6,000$$

Find the production level that will maximize the profit, the maximum profit, and the price that the company needs to charge at the level.

Sol<sup>n</sup> Revenue function is  $R(x) = x(p(x)) = x(200 - \frac{x}{50}) = 200x - \frac{x^2}{50}$

Profit function is  $f(x) = R(x) - C(x) = 200x - \frac{x^2}{50} - (60,000 + 60x) = 140x - \frac{x^2}{50} - 60,000$

The critical number is  $f'(x) = 140 - \frac{2x}{50}$

$$\frac{2x}{50} = 140$$

$$2x = 140(50) \rightarrow x = \frac{140(50)}{2} = 3,500$$

End points

x	f(x)
0	-60000
3500	$490000 - 245000 - 60000 = 185000$
6000	$840000 - 720000 - 60000 = 60000$

$$p(x) = 200 - x$$

$$p(3500) = 200 - \frac{3500}{50} = 200 - 70 = 130$$

$\therefore$  the maximize profit occurs at  $x = 3500$

We should sell 3500 TV sets at the price 130 dollar.

The maximize profit is 185,000 #