

① Let $f(x) = x^2 - 3x$

a) Use the definition to find the derivative of $f(x)$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \rightarrow 0} \frac{[(x+h)^2 - 3(x+h)] - (x^2 - 3x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 3h}{h} = \lim_{h \rightarrow 0} 2x + h - 3 \\ &= 2x - 3 \end{aligned}$$

b) Find the equations of tangent lines of $f(x)$ at $x=0, x=2$ and $x=3$

$y = ax + b$ $f'(x)$ ມີ slope ແລ້ວໄດ້ສຳເນົາ

slope ↗

m a slope ກ່ອນສຳເນົາ $x=0 \rightarrow f'(0) = 2(0) - 3 = -3 \therefore a = -3$

" $x=2 \rightarrow f'(2) = 2(2) - 3 = 1 \therefore a = 1$

" $x=3 \rightarrow f'(3) = 2(3) - 3 = 3 \therefore a = 3$

m b $f(0) = 0^2 - 3(0) = 0$ ໃນ $(0,0)$ $y = -3x + b$ $\left. \begin{array}{l} 0 = -3(0) + b \\ b = 0 \end{array} \right\} \Rightarrow y = -3x + 0 = -3x$

$f(2) = 2^2 - 3(2) = -2$ ໃນ $(2, -2)$ $y = 1x + b$ $\left. \begin{array}{l} -2 = 1(2) + b \\ b = -4 \end{array} \right\} \Rightarrow y = 1x - 4 = x - 4$

$f(3) = 3^2 - 3(3) = 0$ ໃນ $(3, 0)$ $y = 3x + b$ $\left. \begin{array}{l} 0 = 3(3) + b \\ b = -9 \end{array} \right\} \Rightarrow y = 3x - 9$

② Find

a) $\frac{d}{dx}(x^{-5}) = -5x^{-5-1}$
 $= -5x^{-6}$

b) $\frac{d}{dx}\left(\frac{1}{x^5}\right) = \frac{d}{dx}(x^{-5})$
 $= \frac{-5}{x^6}$

$$c) \frac{d}{dx} (x^{\frac{2}{3}}) = \frac{2}{3} x^{-\frac{1}{3}}$$

$$d) \frac{d}{dx} (\sqrt{x}) = \frac{d}{dx} (x^{\frac{1}{2}}) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$e) \frac{d}{dx} \left(\frac{1}{\sqrt{x}} \right) = \frac{d}{dx} \left(x^{-\frac{1}{2}} \right) = \frac{d}{dx} (x^{-\frac{1}{2}}) = -\frac{1}{2} x^{-\frac{3}{2}} = -\frac{1}{2\sqrt{x^3}}$$

$$f) \frac{d}{dx} (x^8 + x^2) = 8x^7 + 2x$$

$$h) \frac{d}{dx} \left(\frac{1}{x} + 4\sqrt{x} \right) = \frac{d}{dx} \left(x^{-1} + 4x^{\frac{1}{2}} \right) = -x^{-2} + 4x^{\frac{1}{2}} = -\frac{1}{x^2} + \frac{4}{\sqrt{x}}$$

$$g) \frac{d}{dx} (6x^{11} - 9) = 66x^{10}$$

$$i) \frac{d}{dx} \left(\frac{x^3}{18} - 4x + 9 \right) = \frac{3x^2}{18} - 4 = \frac{x^2}{6} - 4$$

$$j) y' \text{ for } y = 5x^{-2} + 9x^{-1} \\ y' = 10x^{-3} - 9x^{-2}$$

$$k) h(x) \text{ for } h(x) = \frac{1}{\sqrt{x}} - 2x^{-2} + 4x \\ h(x) = \frac{d}{dx} x^{-\frac{1}{2}} - 2x^{-2} + 4x \\ = -\frac{1}{2} x^{-\frac{3}{2}} + 4x^{-3} + 4 \\ = -\frac{1}{2\sqrt{x^3}} + 4x^{-3} + 4$$

$$l) \frac{dy}{dt} \text{ for } y = \frac{3t^2}{2} - \frac{7}{3t^2} - 3t^{0.2} \\ = \frac{d}{dt} \frac{3t^2}{2} - \frac{7}{3t^3} - 3t^{0.2} \\ = \frac{6t}{2} + \frac{21t^{-4}}{3} - 0.6t^{-0.8} \\ = 3t + \frac{7}{t^4} - 0.6t^{-0.8}$$

$$m) \frac{d}{du} (5u^3 - 4u^{22} + 3u^{\frac{3}{2}} - 5u^{\frac{1}{3}}) \\ = 15u^2 - 8.8u^{1.2} + \frac{9}{2}u^{\frac{1}{2}} - \frac{5}{3}u^{-\frac{2}{3}}$$

$$n) \frac{d}{du} (5\sqrt{u} + \frac{7}{5u^2} + 4) \\ = \frac{d}{du} \left(5u^{\frac{1}{3}} + \frac{7}{5}u^{-2} + 4 \right) \\ = \frac{5}{3}u^{-\frac{2}{3}} - \frac{14}{5}u^{-3}$$

$$= \frac{5}{3\sqrt[3]{u^2}} - \frac{14}{5u^3}$$

③ Let $f'(2) = 3$ and $g'(2) = -1$. Find $h'(2)$ for each $h(x)$ as the following

a) $h(x) = 4f(x)$

$$\cancel{h'(x)} = 4f'(x)$$

$$\cancel{h'(2)} = 4f'(2)$$

$$= 4(3)$$

$$= 12$$

b) $h(x) = f(x) + g(x)$

$$\cancel{h'(x)} = f'(x) + g'(x)$$

$$\cancel{h'(2)} = f'(2) + g'(2)$$

$$= 3 + (-1)$$

$$= 2$$

c) $h(x) = 2f(x) - 3g(x) + 7$

$$\cancel{h'(x)} = 2f'(x) - 3g'(x) + 7$$

$$\cancel{h'(2)} = 2f'(2) - 3g'(2) + 7$$

$$= 2(3) - 3(-1) + 7$$

$$= 6 + 3 + 7 = 16$$

d) $h(x) = g(x) - f(x)$

$$\cancel{h'(x)} = g'(x) - f'(x)$$

$$\cancel{h'(2)} = g'(2) - f'(2)$$

$$= -1 - 3$$

$$= -4$$

④ An object moves along the y axis (marked in feet) so that its position at time x (in seconds) is $y = f(x) = x^3 - 6x^2 + 9x$

a) Find the instantaneous velocity function v .

$$v(x) = f'(x)$$

$$f'(x) = 3x^2 - 12x + 9$$

b) Find the velocities at $x = 2$ and $x = 5$ seconds

$$v(x) = f'(x) = 3x^2 - 12x + 9$$

$$v(2) = f'(2) = 3(2)^2 - 12(2) + 9 = 12 - 24 + 9 = -3$$

$$v(5) = f'(5) = 3(5)^2 - 12(5) + 9 = 75 - 60 + 9 = 24$$

c) Find the time(s) when the velocity is 0

$$v(x) = 0$$

$$3x^2 - 12x + 9 = 0$$

$$3(x^2 - 4x + 3) = 0$$

$$3(x-3)(x-1) = 0$$

$$x = 1, 3$$

⑤ Let $f(x) = \sqrt{x} + 3$. Evaluate dy for

a) $x = 4$; $dx = 0.1$ $dy = f'(x)dx$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4} = 0.25$$

b) $x = 1$; $dx = 0.01$ $dy = f'(x)dx$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(1) = \frac{1}{2\sqrt{1}} = \frac{1}{2} = 0.5$$

$$f'(4)dx = (0.25)(0.1) = 0.025$$

~~$$f'(1)dx = (0.5)(0.01) = 0.005$$~~

⑥ A company manufactures and sell x transistor radios per week. If weekly cost and revenue equations (in dollars) are

$$C(x) = 5000 + 2x, R(x) = 10x - \frac{x^2}{1000}, 0 \leq x \leq 8,000$$

find the approximate changes in revenue and profit production is increased from 6000 to 6,010 unit per week

Revenue function = $R(x) = 10 - \frac{2x}{1000} = 10 - \frac{x}{500}$

marginal Revenue $\downarrow R'(6000) = 10 - \frac{6000}{50} = 10 - 12 = -2$

$\therefore 10 \times (-2) = -20$ means an additional 20 dollars.

Profit function = $R(x) - C(x)$

$$P(x) = 10 - \frac{x}{500} - 2 = 8 - \frac{x}{500}$$

$$P'(6000) = 8 - \frac{6000}{500} = -4$$

$\therefore 10 \times (-4) = -40$ means an additional 40 dollars.

⑦ A company manufactures bottled water. The total cost and revenue (in baht) for the production and sale of x bottles are given, respectively, by

$$C(x) = 5000 + 0.56x, R(x) = \frac{60,000x - x^2}{20,000}$$

a) Find the marginal revenue at $x = 20,000$

Revenue function is $R'(x) = \frac{60,000 - 2x}{20,000}$

$$R'(20,000) = \frac{60,000 - 2(20,000)}{20,000} = \frac{20,000}{20,000}$$

$$= 1$$

b) Find the marginal profit at $x = 20,000, 24,400$ and $30,000$

Profit function is $P(x) = R(x) - C(x)$

$$= \frac{60,000 - 2x}{20,000} - 0.56$$

$$\text{at } x = 20,000 \rightarrow P(20,000) = 1 - 0.56 = 0.44$$

$$\text{at } x = 24,400 \rightarrow P(24,400) = 0.56 - 0.56 = 0$$

$$\text{at } x = 30,000 \rightarrow P(30,000) = 0 - 0.56 = -0.56$$

⑧ The price - demand equation and the cost function for the production of television

sets are given by $p(x) = 300 - \frac{x}{30}$, $C(x) = 150,000 + 30x$

where x is the number of set that can be sold at a price of p per set,
and $C(x)$ is the total cost of producing x set.

a) Find the marginal cost and the marginal revenue

$$C(x) = 30$$

$$\begin{aligned} R(x) &= 300 - \frac{2x}{30} \\ &= 300 - \frac{x}{15} \end{aligned}$$

b) Find $R'(1,500)$ and interpret the results.

$$\begin{aligned} R'(1,500) &= 300 - \frac{15000}{15} \\ &= 300 - 100 = 200 \therefore \text{from 1,500 units to 1,515 units 200 units} \end{aligned}$$

c) Find the profit function in terms of x

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= \left(300x - \frac{x^2}{30}\right) - (150,000 + 30x) \\ &= 270x - 150,000 - \frac{x^2}{30} \end{aligned}$$

d) Find the marginal profit. $P'(1500)$ and interpret the results

$$\begin{aligned} P'(x) &= 270 - \frac{2x}{30} \\ P'(1500) &= 270 - \frac{1500}{15} \\ &= 270 - 100 \\ &= 170 \end{aligned}$$

\therefore from 1,500 units to 1,515 units 170 units

(9) Find y' for

a) $y = \frac{e^x}{2}$

$y' = \frac{e^x}{2}$

b) $y = 4^x + 3e^x$

$y' = 4^x \ln 4 + 3e^x$

c) $y = 5 \ln x$

$y' = \frac{5}{x}$

d) $y = x^2 + \log_{\frac{1}{4}} x + \ln 10$

$y' = 2x + \frac{1}{x \ln 4}$

e) $y = \frac{x^2 - 1}{x^4 + 1}$

$y' = \frac{(x^4 + 1)(2x) - (x^2 - 1)(4x)}{(x^4 + 1)^2}$

f) $y = \ln \left(\frac{x^2 + 1}{e^x} \right)$

$y' = \left(\frac{e^x}{x^2 + 1} \right) \cdot \left[\frac{e^x(2x) - (x^2 + 1)e^x}{(e^x)^2} \right]$

g) $y = \log_2(2x^{\frac{1}{2}} + 3)(4^x)$

$y' = \log_2(2x^{\frac{1}{2}} + 3)(4^x \ln 4) + 4^x \left[\frac{34x}{(2x^{\frac{1}{2}} + 3)\ln 2} \right]$

h) $y = (4x^2 - 1)(7x^3 + x)$

$y' = (4x^2 - 1)(21x^2 + 1) + (7x^3 + x)(8x)$

(10) Use the chain rule to find $\frac{df}{dx}$ for $cu = \ln u$; $u(x) = x^2 + 1$.

$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

$= \frac{1}{u} \cdot 2x = \frac{2x}{u}$

$= \frac{2x}{x^2 + 1}$

(11) Let $y = \frac{3+4u}{1+u^2}$ and $u = 2+x^2$. Find $\frac{dy}{dx}$ and $\frac{dy}{dx}|_{x=1}$

$\frac{dy}{dx}|_{x=1} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \frac{(1+u^2)(4) - (3+4u)(2u)}{(1+u^2)^2} \cdot (2x)$

$= \frac{1+(2+x^2)^2(4) - (3+4(2+x^2))(2)(2+x^2)^2}{[1+(2+x^2)^2]^2} \cdot 2(1)$

$= \frac{40 - 15(6)}{100} \cdot 2 = \frac{40 - 90}{50} = -\frac{50}{50}$
 $= -1$

(12) Find $\frac{dy}{dx}$ for

a) $y = \frac{1}{(x^2 - x + 1)^9}$

$y = (x^2 - x + 1)^{-9}$

$\frac{dy}{dx} = -9(x^2 - x + 1)^{-10}(5x^4 - 1)$

$= -9(5x^4 - 1)$
 $(x^2 - x + 1)^{10}$

b) $y = \sqrt{x^3 - 2x + 5}$

$y = (x^3 - 2x + 5)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(x^3 - 2x + 5)^{-\frac{1}{2}}(3x^2 - 2)$

c) $y = e^{x^3 + 2x^2 + 5}$

$$\frac{dy}{dx} = e^{x^3 + 2x^2 + 5} \cdot (3x^2 + 2x)$$

d) $y = \ln(3x^2 + 5x)$

$$\frac{dy}{dx} = \frac{1}{3x^2 + 5x} \cdot (6x + 5)$$

e) $y = 2^{1-4x^2}$

$$\frac{dy}{dx} = 2^{1-4x^2} \ln 2 (-8x)$$

f) $y = \sin(2x)$

$$\frac{dy}{dx} = \cos(2x) \cdot 2$$

g) $y = \log_4(x^6 + 1)$

$$\frac{dy}{dx} = \frac{1}{(x^6 + 1)\ln 4} \cdot (6x^5)$$

h) $f'(x)$ if $f(x) = (1 + \ln x)^{27}$

$$\frac{df}{dx} = 27(1 + \ln x)^{26} \cdot \left(\frac{1}{x}\right)$$

i) $g'(x)$ if $g(x) = 5x e^{3x}$

$$g'(x) = 15x e^{3x} + 5e^{3x}$$

$$j) \frac{d}{dx} \left[\frac{\sin(5x+1)}{1+\cos x} \right]^x$$

$$= \frac{(1+\cos x)[5\cos(5x+1)] - \sin(5x+1)[-5\sin x]}{(1+\cos x)^2}$$

k) $\frac{d}{dx} \left[\frac{(x^3+2)^5}{2x^8} \right]$

l) $\frac{d}{dt} 3(2t^2 + t)^{-5}$

$$= \frac{30x^{10}(x^3+2)^4 - (x^3+2)^5(16x^7)}{(2x^8)^2}$$

$$= -15(2t^2 + t)^{-6}(4t+1)$$

(13) Find the slope of the tangent line to the graph of each $f(x)$ at the given point x .

a) $f(x) = (2x+5)^3$; $x=1$

$$f'(x) = 3(2x+5)^2(2)$$

$$= 6(2x+5)^2$$

$$f'(1) = 6(2(1)+5)^2$$

$$= 6(7)^2$$

$$= 234$$

b) $f(x) = \sqrt{\ln x}$; $x=e$

$$f'(x) = \frac{1}{2} (\ln x)^{-\frac{1}{2}} \left(\frac{1}{e}\right)$$

$$= \frac{1}{2} \cdot \frac{1}{e}$$

$$= \frac{1}{2e}$$

c) $f(x) = \frac{x^4}{(3x-8)^2}$; $x=4$

$$f'(x) = \frac{(3x-8)^2(4x^3) - (x^4)[2(3x-8)(3)]}{(3x-8)^4}$$

$$= \frac{(3x-8)[(3x-8)(4x^3) - (x^4)(2)(3)]}{(3x-8)^4}$$

$$f'(4) = \frac{1024 - 1536}{64}$$

$$= -8$$

(14) Find the following limits if exist

$$\text{a) } \lim_{x \rightarrow \infty} \frac{x^2}{\ln x} = \lim_{x \rightarrow \infty} \frac{\frac{2x}{1}}{\frac{1}{x}}$$

$$= \lim_{x \rightarrow \infty} 2x(x)$$

$$= \infty \infty$$

$$\text{b) } \lim_{x \rightarrow 0} \frac{2-e^x-e^{-x}}{x^2} = \lim_{x \rightarrow 0} \frac{-e^x-e^{-x}(-1)}{2x}$$

$$= \lim_{x \rightarrow 0} \frac{-e^x + \frac{1}{e^x}}{2x}$$

$$= \lim_{x \rightarrow 0} \frac{-e^{2x} + 1}{e^x \cdot 2x}$$

$$= \lim_{x \rightarrow 0} \frac{-e^{2x} + 1}{(e^x)(2x)} = \lim_{x \rightarrow 0} \frac{-2e^{2x}}{2xe^x + 2e^x}$$

$$= \frac{-2e^0}{2(0)e^0 + 2e^0} = \frac{-2(1)}{2(1)} = -1$$

(15) Use implicit differentiation to find $\frac{dy}{dx}$ for

$$\text{a) } 4x^2 + 4y - y^3 = 4$$

$$\frac{d}{dx}(4x^2 + 4y - y^3) = \frac{d}{dx}(4)$$

$$8x + 4\frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(4 - 3y^2) = -8x$$

$$\frac{dy}{dx} = \frac{-8x}{4 - 3y^2}$$

$$\text{b) } y - xy^2 + x^2 + 1 = 0$$

$$\frac{d}{dx}(y - xy^2 + x^2 + 1) = 0$$

$$\frac{dy}{dx} \left(x^2y \frac{dy}{dx} + y^2 \frac{d}{dx} \right) + 2x = 0$$

$$\frac{dy}{dx}(1 - 2xy) = -2x + y^2$$

$$\frac{dy}{dx} = \frac{-2x + y^2}{1 - 2xy}$$

$$\text{c) } xe^x + \ln y - 3y = 0$$

$$\frac{d}{dx}(xe^x + \ln y - 3y) = 0$$

$$xe^x + e^x + \cancel{y} \cdot \frac{dy}{dx} - 3 \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} \left(\frac{1}{y} - 3 \right) = -xe^x - e^x$$

$$\frac{dy}{dx} = \frac{-xe^x - e^x}{\frac{1}{y} - 3}$$

$$\text{d) } 6\sqrt{y^3+1} - 2\cos 3x - 1 = 0$$

$$\frac{d}{dx}(6\sqrt{y^3+1} - 2\cos 3x - 1) = 0$$

$$3(y^3+1)^{\frac{1}{2}}(3y^2) \frac{dy}{dx} + 2\sin 3x(3) = 0$$

$$\frac{dy}{dx} \cdot \frac{9y^2}{\sqrt{y^3+1}} = 6\sin 3x$$

$$\frac{dy}{dx} = \frac{-6\sin(3x)(\sqrt{y^3+1})}{9y^2}$$

(16) Find the slope of the tangent line to the graph of the equation $y - xy^2 + x^2 + 1 = 0$ at $x=1$

$$\frac{d}{dx}(y - xy^2 + x^2 + 1) = 0 \quad | \quad (x, y) = (1, 2)$$

$$\frac{dy}{dx} - (x^2y \frac{dy}{dx} + y^2) + 2x = 0 \quad | \quad \frac{dy}{dx}|_{(1,2)} = \frac{2x + y^2}{1 - 2xy} = \frac{-2(1) + (2)^2}{1 - 2(1)(2)}$$

$$\frac{dy}{dx}(1 - 2xy) = -2x + y^2 \quad | \quad = \frac{-2 + 4}{1 - 4} = -\frac{2}{3}$$

$$\frac{dy}{dx} = \frac{-2x + y^2}{1 - 2xy} \quad | \quad \frac{dy}{dx}|_{(1,-1)} = \frac{-2x + y^2}{1 - 2xy} = \frac{-2(1) + (-1)^2}{1 - 2(1)(-1)}$$

$$x=1$$

$$y - (1)2 + (1)^2 + 1 = 0$$

$$y - y^2 + 2 = 0$$

$$(y-2)(y+1) = 0$$

$$y = 2, -1$$

$$= \frac{-2+1}{1+2} = -\frac{1}{3}$$

(17) Use properties of exponential and logarithmic functions to find y' for

$$a) y = e^{x^2+x+1} (x^3+6)^5 (x^2+3)^{\frac{3}{2}}$$

$$\ln y = \ln [e^{x^2+x+1} (x^3+6)^5 (x^2+3)^{\frac{3}{2}}]$$

$$\ln y = (x^2+x+1) + 5\ln(x^3+6) + \frac{3}{2}\ln(x^2+3)$$

$$\frac{d}{dy}(\ln y) = \frac{d}{dx} [(x^2+x+1) + 5\ln(x^3+6) + \frac{3}{2}\ln(x^2+3)]$$

$$\frac{1}{y} \left(\frac{dy}{dx} \right) = 2x+1 + 5 \cdot \frac{1}{x^3+6} (3x^2) + \frac{3}{2} \cdot \frac{1}{x^2+3} (2x)$$

$$\frac{1}{y} \left(\frac{dy}{dx} \right) = 2x+1 + \frac{15x^2}{x^3+6} + \frac{3x}{x^2+3}$$

$$\frac{dy}{dx} = \left[e^{x^2+x+1} (x^3+6)^5 (x^2+3)^{\frac{3}{2}} \right] \left[2x+1 + \frac{15x^2}{x^3+6} + \frac{3x}{x^2+3} \right]$$

$$b) y = \frac{(10-4e^x)(2x^5+1)^{10}}{\sqrt{x^2+4x+5}}$$

$$\ln y = \frac{\ln(10-4e^x)(2x^5+1)^{10}}{\sqrt{x^2+4x+5}} = \ln(10-4e^x) + 10\ln(2x^5+1) - \frac{1}{2}\ln(x^2+4x+5)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx} (\ln(10-4e^x) + 10\ln(2x^5+1) - \frac{1}{2}\ln(x^2+4x+5))$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1(-4e^x)}{10-4e^x} + 10 \left(\frac{1}{2x^5+1} \right) (10x^4) - \frac{1}{2} \left(\frac{1}{x^2+4x+5} \right) (2x+4)$$

$$\frac{dy}{dx} = \frac{(10-4e^x)(2x^5+1)^{10}}{\sqrt{x^2+4x+5}} \left(\frac{-4e^x}{10-4e^x} + \frac{100x^4}{2x^5+1} + \frac{x+2}{x^2+4x+5} \right)$$

(18) Find $\frac{dy}{dx}$ for $xe^x + \ln y - 3y = 0$

$$\frac{dy}{dx} = xe^x + e^x + \frac{1}{y} \left(\frac{dy}{dx} \right) - 3 \left(\frac{dy}{dx} \right) = 0$$

$$\frac{dy}{dx} = \frac{-xe^x - e^x}{\frac{1}{y} - 3}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} \left(\frac{-xe^x - e^x}{\frac{1}{y} - 3} \right)$$

$$= \left(\frac{1}{y} - 3 \right) (-xe^x - e^x) - (-xe^x - e^x) \left(\frac{-1}{y^2} \right) \left(\frac{dy}{dx} \right)$$

$$= \left(\frac{1}{y} - 3 \right) (-xe^x - 2e^x) - (xe^x + e^x) \left(\frac{1}{y^2} \right) \left(\frac{-xe^x - e^x}{\frac{1}{y} - 3} \right)$$

$$\left(\frac{1}{y} - 3 \right)^2$$

(19) Find the Taylor polynomial for $f(x) = x^{10}$ about $x=1$, and use $p_2(x)$ to estimate $(1.01)^{10}$

Taylor ; $f(x) = x^{10}, x=1$

$$P_2(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!}$$

$$= f(1) + f'(1)(x-1) + \frac{f''(1)(x-1)^2}{2!}$$

$$f(x) = x^{10} \quad | \quad f'(x) = 10x^9 \quad | \quad f''(x) = 90x^8$$

$$f(1) = 1 \quad | \quad f'(1) = 10 \quad | \quad f''(1) = 90$$

$$P_2(x) = 1 + 10(x-1) + \frac{90(x-1)^2}{2}$$

$$f(x) \approx P_2(x)$$

$$(1.01)^{10} = f(1.01) \approx P_2(1.01)$$

$$\approx 1 + 10(1.01-1) + 45(1.01-1)^2$$

$$\approx 1 + 0.1 + 0.0045$$

$$\approx 1.1045$$

$$\therefore (1.01)^{10} \approx p_2(1.01) = 1.1045$$