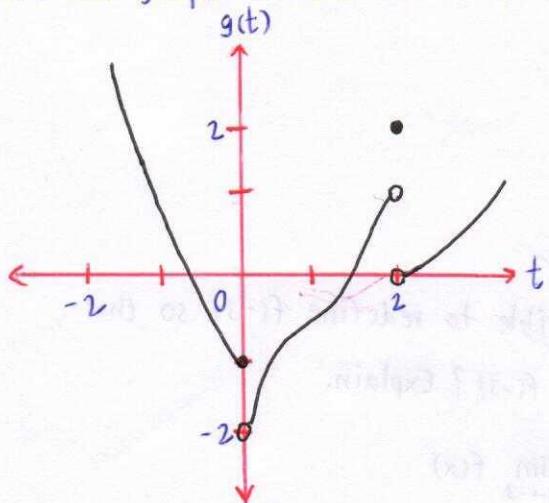


EXERCISES

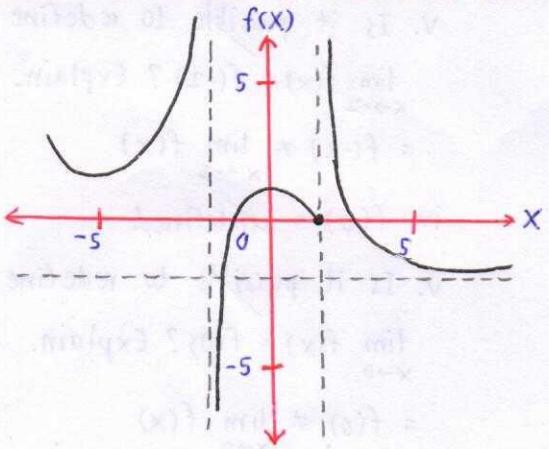
$$\frac{82}{3} = 27.3$$

1. Use the graph of the function $g(t)$ to estimate the indicated function values and limits.



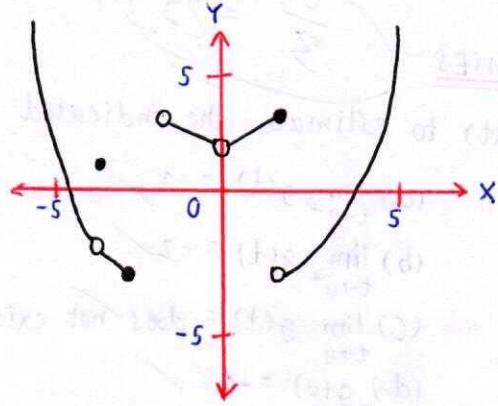
- (a) $\lim_{t \rightarrow 0^-} g(t) = -1$
- (b) $\lim_{t \rightarrow 0^+} g(t) = -2$
- (c) $\lim_{t \rightarrow 0} g(t) = \text{does not exist}$
- (d) $g(0) = -1$
- (e) $\lim_{t \rightarrow 2^-} g(t) = 1$
- (f) $\lim_{t \rightarrow 2^+} g(t) = 0$
- (g) $\lim_{t \rightarrow 2} g(t) = \text{does not exist}$
- (h) $g(2) = 2$

2. Use the graph to estimate the indicated function values and limits.



- (a) i. $\lim_{x \rightarrow \infty} f(x) = \infty$
ii. $\lim_{x \rightarrow -\infty} f(x) = \infty$
- (b) i. $\lim_{x \rightarrow 2^-} f(x) = 0$
ii. $\lim_{x \rightarrow 2^+} f(x) = \infty$
iii. $\lim_{x \rightarrow 2} f(x) = \text{does not exist}$
iv. Is f continuous at $x=2$? = discontinuous at $x=2$
- (c) i. $\lim_{x \rightarrow -1^-} f(x) = 0$
ii. $\lim_{x \rightarrow -1^+} f(x) = 0$
iii. $\lim_{x \rightarrow -1} f(x) = 0$
iv. Is f continuous at $x=-1$? = continuous at $x=-1$
- (d) i. $\lim_{x \rightarrow -2^-} f(x) = \infty$
ii. $\lim_{x \rightarrow -2^+} f(x) = -\infty$
iii. $\lim_{x \rightarrow -2} f(x) = \text{does not exist}$
iv. Is f continuous at $x=-2$? = discontinuous at $x=-2$

3. Use the graph to estimate the indicated function values and limits.



(a) i. $\lim_{x \rightarrow -3^+} f(x) = -2$

ii. $\lim_{x \rightarrow -3^-} f(x) = -2$

iii. $\lim_{x \rightarrow -3} f(x) = -2$

iv. $f(-3) = 1$

v. Is it possible to redefine $f(-3)$ so that $\lim_{x \rightarrow -3} f(x) = f(-3)$? Explain.

$$= f(-3) \neq \lim_{x \rightarrow -3} f(x)$$

iv. $f(-2) = -3$

v. Is it possible to redefine $f(-2)$ so that $\lim_{x \rightarrow -2} f(x) = f(-2)$? Explain.

$$= f(-2) \neq \lim_{x \rightarrow -2} f(x)$$

iv. $f(0) = \text{undefined}$

v. Is it possible to redefine $f(0)$ so that $\lim_{x \rightarrow 0} f(x) = f(0)$? Explain.

$$= f(0) \neq \lim_{x \rightarrow 0} f(x)$$

iv. $f(2) = 3$

v. Is it possible to redefine $f(2)$ so that $\lim_{x \rightarrow 2} f(x) = f(2)$? Explain.

$$= f(2) \neq \lim_{x \rightarrow 2} f(x)$$

4. Find each limit.

(a) $\lim_{x \rightarrow 3} (5x^3 + 4) = 5(3)^3 + 4 = 139$

(b) $\lim_{x \rightarrow -1} (x^4 - 2x + 3) = (-1)^4 - 2(-1) + 3 = 1 + 2 + 3 = 6$

(c) $\lim_{x \rightarrow 1} \frac{4}{3x^2 - 5} = \frac{4}{3(1)^2 - 5} = \frac{4}{-2} = -2$

(d) $\lim_{x \rightarrow -2} \frac{x^2}{x^2 + 1} = \frac{(-2)^2}{(-2)^2 + 1} = \frac{4}{5}$

(e) $\lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{x - 3} = \frac{(x-3)(x-3)}{(x-3)} = x - 3 = 3 - 3 = 0$

(f) $\lim_{x \rightarrow 2^-} \frac{3x^2 - x - 10}{x^2 - 4} = \frac{(3x+5)(x-2)}{(x+2)(x-2)} = \frac{3x+5}{x+2} = \frac{3(2)+5}{2+2} = \frac{11}{4}$

(g) $\lim_{x \rightarrow 1^+} \frac{x^4 - 1}{x-1} = \frac{(x^2 + 1)(x+1)(x-1)}{(x-1)} = (x^2 + 1)(x+1) = (1^2 + 1)(1+1) = 2 \cdot 2 = 4$

$$(h) \lim_{x \rightarrow 5} \frac{x^2 - 3x - 10}{x^2 - 10x + 25} = \frac{(x-5)(x+2)}{(x-5)(x-5)} = \frac{x+2}{x-5} = \frac{5+2}{5-5} = \frac{7}{0} = \text{does not exist}$$

$$(i) \lim_{x \rightarrow 2^-} \frac{|5x-10|}{x-2} = \frac{-(5x-10)}{x-2} = \frac{-5(x-2)}{x-2} = -5$$

$$(j) \lim_{x \rightarrow 2^+} \frac{|5x-10|}{x-2} = \frac{5x-10}{x-2} = \frac{5(x-2)}{x-2} = 5$$

$$(k) \lim_{x \rightarrow 2} \frac{|5x-10|}{x-2} = \text{does not exist (from (i), (j))}$$

$$(l) \lim_{x \rightarrow \infty} (2^{12} - 20x^{15}) = 2^{12} - 20\infty^{15} = 2^{12} - \infty = -\infty$$

$$(m) \lim_{x \rightarrow -\infty} (-17x^{-1}) = \lim_{x \rightarrow -\infty} \left(\frac{1}{-17x} \right) = \frac{1}{(-17)(-\infty)} = \frac{1}{\infty} = 0$$

$$(n) \lim_{x \rightarrow -\infty} \frac{5x^3 - 2x^2 + 1}{3x^3 + 5} = \frac{5}{3}$$

$$(o) \lim_{x \rightarrow -\infty} \frac{7x^2 + x + 11}{4 - x^4} = \infty$$

$$(p) \lim_{x \rightarrow -\infty} \frac{9x^2 + x + 3}{2 - x} = 0$$

$$(q) \lim_{x \rightarrow \infty} e^{5x} = e^{5(\infty)} = e^{\infty} = \infty$$

$$(r) \lim_{x \rightarrow -\infty} e^{-x} = e^{-(-\infty)} = e^{\infty} = \infty$$

$$(s) \lim_{x \rightarrow \infty} 7^{\frac{1}{2}x} = 7^{\frac{1}{2}\infty} = 7^{\infty} = \infty$$

$$(t) \lim_{x \rightarrow \infty} \ln 8 = \ln 8$$

$$(u) \lim_{x \rightarrow -\infty} 3^{3x} = 3^{3(-\infty)} = 3^{-\infty} = \frac{1}{3^\infty} = \frac{1}{\infty} = 0$$

5. Let $f(x) = \begin{cases} 1-x^2 & \text{if } x \leq 1, \\ 1+x^2 & \text{if } x > 1. \end{cases}$ Find

$$(a) \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 1+x^2 = 1+0^2 = 1$$

$$(b) \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} 1-x^2 = 1-0^2 = 1$$

$$(c) \lim_{x \rightarrow 0} f(x) = 1 \text{ (from a, b)}$$

$$(d) f(0) = f(x) = 1-x^2 \\ \therefore f(0) = 1-0^2 = 1$$

6. Let $f(x) = \frac{|x-1|}{x-1}$. Find

$$(a) \lim_{x \rightarrow 1^+} f(x) = \frac{x-1}{x-1} = 1$$

$$(b) \lim_{x \rightarrow 1^-} f(x) = \frac{-(x-1)}{x-1} = -1$$

$$(c) \lim_{x \rightarrow 1} f(x) = \text{does not exist} \\ (\text{from a, b})$$

$$(d) f(1) = \frac{0}{0} = \text{undefined}$$

7. Evaluate $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$ when

(a) $f(x) = 3x + 1$

Soln

$$f(x) = 3x + 1$$

$$f(x+h) = 3(x+h) + 1 = 3x + 3h + 1$$

$$f(x+h) - f(x) = (3x + 3h + 1) - (3x + 1)$$

$$= 3x + 3h + 1 - 3x - 1$$

$$= 3h$$

$$\frac{f(x+h) - f(x)}{h} = \frac{3h}{h} = 3$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} 3 = 3$$

$$\therefore \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = 3$$

(b) $f(x) = x^2 - 2$

Soln

$$f(x) = x^2 - 2$$

$$f(x+h) = (x+h)^2 - 2 = x^2 + 2xh + h^2 - 2$$

$$f(x+h) - f(x) = (x^2 + 2xh + h^2 - 2) - (x^2 - 2)$$

$$= x^2 + 2xh + h^2 - 2 - x^2 + 2$$

$$= 2xh + h^2$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2}{h} = 2x + h$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} 2x + h = 2x + 0 = 2x$$

$$\therefore \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = 2x$$

8. For each function f , locate all points of discontinuity, and discuss the behavior of f at these points.

(a) $f(x) = \begin{cases} 1+x & \text{if } x < 1, \\ 5-x & \text{if } x \geq 1 \end{cases}$

Soln

$$1. f(1) = 5-1 = 4$$

$$2. \lim_{x \rightarrow 1} f(x) = \left\{ \begin{array}{l} \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (1+x) = 1+1 = 2 \\ \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (5-x) = 5-1 = 4 \end{array} \right\} \lim_{x \rightarrow 1} f(x) = \text{does not exist}$$

$$3. f(1) \neq \lim_{x \rightarrow 1} f(x)$$

$\therefore f(x)$ is discontinuous at $x=1$

$$(b) f(x) = \begin{cases} x^2 & \text{if } x < 1, \\ 2x & \text{if } x \geq 1 \end{cases}$$

Sol'n 1. $f(1) = 2(1) = 2$

2. $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x^2) = 1^2 = 1$ $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (2x) = 2(1) = 2$ $\lim_{x \rightarrow 1} f(x) = \text{does not exist}$

3. $f(1) \neq \lim_{x \rightarrow 1} f(x)$

$\therefore f(x)$ is discontinuous at $x=1$

$$(c) f(x) = \begin{cases} x^2 & \text{if } x \leq 2, \\ 2x & \text{if } x > 2 \end{cases}$$

Sol'n 1. $f(2) = 2^2 = 4$

2. $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (x^2) = 2^2 = 4$ $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (2x) = 2(2) = 4$ $\lim_{x \rightarrow 2} f(x) = 4$

3. $f(2) = \lim_{x \rightarrow 2} f(x)$

$\therefore f(x)$ is continuous at $x=2$

9. For each function, determine where each function is continuous.

(a) $f(x) = 3x - 4$ (continuous)

(b) $g(x) = \frac{x-2}{(x-3)(x+1)}$ (discontinuous at $x=3, -1$)

(c) $h(x) = \frac{1-x^2}{1+x^2}$ (continuous)

(d) $j(x) = \frac{x^2+4}{4-25x^2}$ (discontinuous at $x=$)

10.

Let $f(x) = \begin{cases} x^2 & \text{if } x \leq -3, \\ 3a & \text{if } -3 < x \leq 3, \\ b-5x & \text{if } x > 3. \end{cases}$

(a) If $a=2$ and $b=6$, find the values of x where function f is discontinuous.

Sol'n $f(x) = \begin{cases} 6 & \text{if } -3 < x \leq 3 \\ 6-5x & \text{if } x > 3 \end{cases}$

1. $f(3) = 6$

2. $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} 6$ $\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} 6-5x$ thus $6 \neq 6-5x$
 $5x \neq 0$
 $x \neq 0$

\therefore the values of x is $\mathbb{R}-\{0\}$ so that $f(x)$ is discontinuous.

(b) If $a=3$, find the value of b so that f is continuous.

Soln $f(x) = \begin{cases} 9 & \text{if } -3 < x \leq 3, \\ b-5x & \text{if } x > 3 \end{cases}$

1. $f(3) = 9$

2. $\lim_{x \rightarrow 3} f(x) = \left\{ \begin{array}{l} \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} 9 = 9 \\ \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} b-5x = b-5(3) = b-15 \end{array} \right\} \lim_{x \rightarrow 3} f(x) = 9$

thus $9 = b-15$
 $b = 24$

\therefore the value of b is 24 so that $f(x)$ is continuous.