

Midterm ข้อ 2 1/2561

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$$1. z_1 = 3 \left(\cos\left(\frac{3\pi}{5}\right) + i \sin\left(\frac{3\pi}{5}\right) \right) \quad \left| \quad z_2 = 2 \left(\cos\left(\frac{23\pi}{20}\right) + i \sin\left(\frac{23\pi}{20}\right) \right) \right.$$

$$|z_1| = r_1 = 3 \quad \theta_1 = \frac{3\pi}{5} = \arg(z_1) \quad \left| \quad |z_2| = r_2 = 2, \quad \theta_2 = \frac{23\pi}{20} = \arg(z_2) \right.$$

$$1.1 \quad \underset{\substack{\uparrow \\ \text{modulus}}}{|z_2(z_1)^4|} = |z_2| |z_1|^4 = 2 \cdot 3^4 = 162$$

$$\arg(z_2 z_1^4) = \arg(z_2) + \arg(z_1^4) = \arg(z_2) + 4 \arg(z_1)$$

= \uparrow $\arg(z_1)$ คูณกับ 4 อดส์

$$= \frac{3\pi}{5} + 4 \left(\frac{23\pi}{20} \right) = \frac{3\pi}{5} + \frac{23\pi}{5} = \frac{26\pi}{5} = 4\pi + \frac{\pi}{5} + \frac{\pi}{5}$$

$$\text{Arg}(z_2 z_1^4) = -\frac{4\pi}{5}$$

$$1.2 \quad \text{หา } z = z_2 z_1^4 \quad \text{หา } |z| = 162, \theta = \text{Arg } z = -\frac{4\pi}{5}$$

$$\text{หา } z = |z| \left(\cos(\theta) + i \sin(\theta) \right)$$

$$= 162 \left(\cos\left(-\frac{4\pi}{5}\right) + i \sin\left(-\frac{4\pi}{5}\right) \right)$$

$$= 162 \left(\cos\left(\frac{4\pi}{5}\right) - i \sin\left(\frac{4\pi}{5}\right) \right)$$

$$a = 162 \cos\left(\frac{4\pi}{5}\right) \quad b = -162 \sin\left(\frac{4\pi}{5}\right)$$

$$= -162 \cos\left(\frac{\pi}{5}\right) \quad b = 162 \sin\left(\frac{\pi}{5}\right) ?$$

อย่าลืมนะ! โคนค่าที่หาของ

ค่า sine หรือ cosine ของ $\frac{\pi}{5}$ ให้หาค่าพวกนี้ไว้ก่อน

ถ้ามีค่าพวกนี้แล้วก็จะทำตัวคูณได้สะดวกขึ้นกับ $\frac{\pi}{3}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{\pi}{4}, \pi, \dots$

$$2. z^2 + (4+i)z + (3+3i) = 0$$

$$z = \frac{-(4+i) \pm \sqrt{(4+i)^2 - 4(3+3i)}}{2} = \frac{-4-i \pm \sqrt{3-4i}}{2}$$

วิธีสอนที่สอน 3-4i (Praxis #1) $|3-4i| = \sqrt{9+16} = 5$ $x=3, y=-4 < 0$

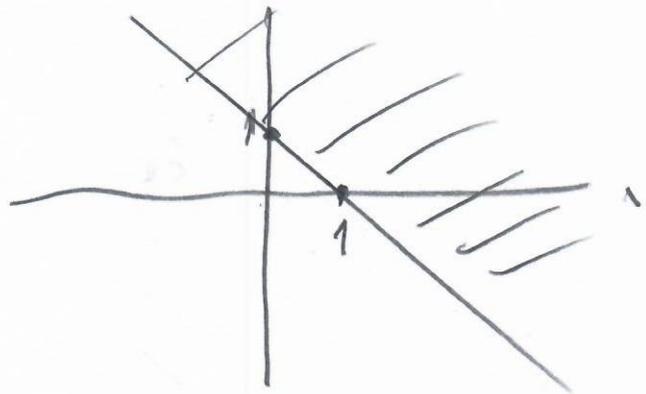
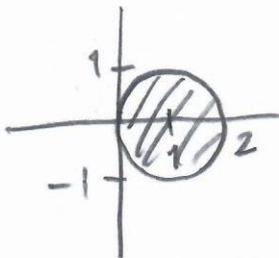
$$w_1 = \sqrt{\frac{5+3}{2}} - i\sqrt{\frac{5-3}{2}} = 2-i$$

$$z_1 = \frac{-4-i+2-i}{2} = \frac{-2-2i}{2} = -1-i \quad \left| \quad z_2 = \frac{-4-i-2+i}{2} = \frac{-6}{2} = -3 \right.$$

$$3.1 \quad |1-z|^2 \leq 1$$

or $|1-z| \leq 1 \quad |-(z-1)| \leq 1$ or $|z-1| \leq 1$

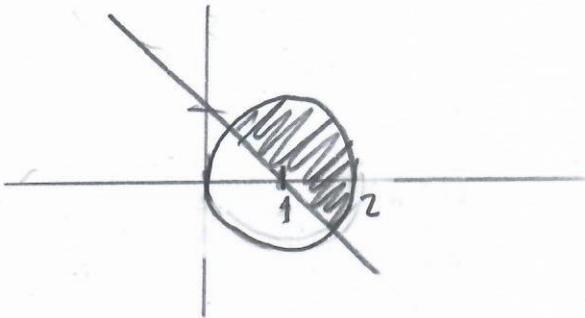
d.d.n = 1
จุด 1



$$3.2 \quad \text{Re}(z) + \text{Im}(z) \geq 1$$

$$x + y \geq 1$$

3.3



$$4.1 \quad \text{Wojna } f(z) = \frac{z^3 + iz^2 - z - i}{(z-i)(z+i)} = \frac{(z+i)(z^2-1)}{(z-i)(z+i)}$$

$$= \frac{z^2-1}{z-i}$$

$$4.1 \quad \lim_{z \rightarrow -i} f(z) = \lim_{z \rightarrow -i} \frac{z^2-1}{z-i} = \frac{(-i)^2-1}{-i-i} = \frac{-2}{-2i} = \frac{1}{i} = -i$$

4.2 $z_0 = -i$

$$\begin{aligned}
 5. \quad f(z) &= \frac{1}{\bar{z}} + |z|^2 + \operatorname{Re}(z+2) - i \\
 &= \frac{1}{x-iy} \cdot \frac{x+iy}{x+iy} + x^2+y^2 + \operatorname{Re}(x+iy+2) - i \\
 &= \frac{x}{x^2+y^2} + i \frac{y}{x^2+y^2} + x^2+y^2 + x+2 - i \\
 &= \left(\frac{x}{x^2+y^2} + x^2+y^2 + x+2 \right) + i \left(\frac{y}{x^2+y^2} - 1 \right)
 \end{aligned}$$

$$\begin{aligned}
 6. \quad f(z) &= e^{2x} \cos 3y - i e^{2x} \sin 3y \\
 u &= e^{2x} \cos 3y & v &= -e^{2x} \sin 3y \\
 u_x &= 2e^{2x} \cos 3y & v_y &= -3e^{2x} \cos 3y \rightarrow u_x \neq v_y
 \end{aligned}$$

↪ не является функцией комплексной переменной

$$\begin{aligned}
 7.1 \quad u &= e^y \cos x + kx^2 + ky^2 - 2xy + x \\
 u_x &= -e^y \sin x + 2kx - 2y + 1 & u_y &= e^y \cos x + 2ky - 2x \\
 u_{xx} &= -e^y \cos x + 2k & u_{yy} &= e^y \cos x + 2k \\
 u_{xx} + u_{yy} &= -e^y \cos x + 2k + e^y \cos x + 2k = 0 \\
 & & 4k &= 0 \\
 & & k &= 0 \\
 & \text{↪ не является функцией комплексной переменной} & k &= 0 \#
 \end{aligned}$$

$$\begin{aligned}
 7.2 \quad u_x &= v_y \quad \text{и наоборот} \quad k=0 \\
 v_y &= -e^y \sin x - 2y + 1 \\
 v(x,y) &= -e^y \sin x - y^2 + y + \phi(x) \\
 v_x &= -e^y \cos x + \phi'(x) = -(e^y \cos x - 2x) \\
 -e^y \cos x + \phi'(x) &= -e^y \cos x + 2x \\
 \phi'(x) &= 2x \\
 \phi(x) &= x^2 + C \\
 \boxed{v(x,y)} &= -e^y \sin x - y^2 + y + x^2 + C
 \end{aligned}$$

Cauchy-Riemann
 ① $u_x = v_y$ ② $u_y = -v_x$
 $v_x = -u_y$

8.1 ให้ $a_n = \frac{3^{2n+1}}{(n+9)!} + 1$ $n=1, 2, \dots$

Anti $-a_n = \frac{3^{2n+3}}{(n+10)!} - \frac{3^{2n+1}}{(n+9)!}$
 $= \frac{3^{2n+1}}{(n+9)!} \left[\frac{3^2 - 1}{n+10} \right]$

พหุคูณ $\left\{ \frac{9}{n+10} \right\}$ เป็นลำดับลด
 เพราะ $\left\{ \frac{9}{n+10} \right\}$ เป็นลำดับลด
 ค่าพหุคูณลด $\frac{9}{11} < 1$

ดังนั้น $\left(\frac{9}{n+10} - 1 \right) < 0$

จึงได้ Anti $-a_n < 0$

$a_{n+1} < a_n$ แสดงว่า $\{a_n\}$ เป็นลำดับลด
 เป็นลำดับที่ลู่เข้า

8.2 $a_n > 0$ แสดงว่า $\{a_n\}$ เป็นลำดับลด

$\frac{9}{11} = a_1 > a_2 > a_3 > \dots > a_n > \dots$

แสดงว่า $\lim_{n \rightarrow \infty} a_n = 0$

ลำดับที่ลู่เข้า

9.1 $\sum_{k=1}^{\infty} \frac{5}{3^k} = \frac{5}{3} + \frac{5}{3} \left(\frac{1}{3} \right) + \frac{5}{3} \left(\frac{1}{3} \right)^2 + \dots = \frac{\frac{5}{3}}{1 - \frac{1}{3}} = \frac{5}{3} \cdot \frac{3}{2} = \boxed{\frac{5}{2}}$
 แสดงว่า $r = \frac{1}{3}$ $a = \frac{5}{3}$

9.2 $\sum_{k=1}^{\infty} \left(\frac{1}{k^2} - \frac{1}{(k+1)^2} \right)$
 $S_n = \left(1 - \frac{1}{2^2} \right) + \left(\frac{1}{2^2} - \frac{1}{3^2} \right) + \left(\frac{1}{3^2} - \frac{1}{4^2} \right) + \left(\frac{1}{4^2} - \frac{1}{5^2} \right) + \dots + \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right)$
 $S_n = 1 - \frac{1}{(n+1)^2}$ $\lim_{n \rightarrow \infty} S_n = 1$

ผลรวมคือ 1

$$9.3 \quad \sum_{k=1}^{\infty} \left(\frac{7}{3^k} + \frac{1}{k^2} - \frac{1}{(k+1)^2} \right) = \sum_{\substack{\text{bsh } r=1/3}} \frac{7}{3^k} + \sum_{\substack{\text{sh } 9.2}} \left(\frac{1}{k^2} - \frac{1}{(k+1)^2} \right)$$

$$= \frac{7/3}{1 - \frac{1}{3}} + 1 = \frac{7}{2} + 1 = \boxed{\frac{9}{2}}$$

10. $\sum \frac{5k^4 - k^2}{2k^6 + k^2 + 1} a_k$

$b_k = \frac{5k^4}{2k^6} = \frac{5}{2k^2}$ $\sum b_k = \frac{5}{2} \sum \frac{1}{k^2}$
auswahl p=2
bsh

$\frac{5k^4}{2k^6 + k^2 + 1} < \frac{5k^4}{2k^6}$ bsh: $2k^6 + k^2 + 1 > 2k^6$

$a_k = \frac{5k^4 - k^2}{2k^6 + k^2 + 1} < \frac{5k^4}{2k^6} = b_k$ bsh: $5k^4 - k^2 < 5k^4$

bshmasa nshu nshu shla $\sum a_k$ shu

11.1 $f(x) = \frac{\ln(4x)}{x^2} \quad x \geq 1$

$$f'(x) = \frac{x^2 \left(\frac{1}{4x} \right) \cdot 4 - \ln(4x)}{x^4}$$

$$= \frac{1 - \ln(4x)}{x^2} < 0$$

$x^2 > 0$

$\ln(4) > 1$
 shu $1 - \ln(4x) < 0$
 shu $x \geq 1$

$\therefore f$ shu shu shu shu

11.2 $\int_1^{\infty} \frac{\ln(4x)}{x} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{\ln(4x)}{x} dx$

$$= \lim_{b \rightarrow \infty} \left[\frac{(\ln(4b))^2}{2} - 0 \right] = \infty$$

shu shu shu shu

$$\int \frac{\ln(ax)}{x} dx = \int \frac{\ln(ax)}{x} d(ax)$$

$$= \int \frac{\ln(ax)}{(ax)} d(ax)$$

$$= \int \ln(ax) d(\ln(ax))$$

$$= \frac{[\ln(ax)]^2}{2} + C$$

12. $\sum_{k=1}^{\infty} \frac{\sqrt{k} - \sin\left(\frac{1}{\sqrt{k}}\right)}{k\sqrt{k}}$, $0 < \sin\left(\frac{1}{\sqrt{k}}\right) \leq 1$

$\sin\left(\frac{1}{\sqrt{k}}\right) \rightarrow 0$
 $k \rightarrow \infty$

$b_k = \frac{\sqrt{k} - 1}{k\sqrt{k}} < \frac{\sqrt{k} - \sin\left(\frac{1}{\sqrt{k}}\right)}{k\sqrt{k}} = a_k$

but $b_k = \frac{\sqrt{k} - 1}{k\sqrt{k}}$ \rightarrow $\frac{\sqrt{k} - 1}{k\sqrt{k}}$ \sum \downarrow ∞

but $c_k = \frac{\sqrt{k}}{k\sqrt{k}} = \frac{1}{k}$ $\sum \frac{1}{k}$ \downarrow ∞ \rightarrow ∞

but $c_k = \frac{\sqrt{k}}{k\sqrt{k}} = \frac{1}{k}$ $\sum \frac{1}{k}$ \downarrow ∞ \rightarrow ∞

$\lim_{k \rightarrow \infty} \frac{b_k}{c_k} = \frac{\sqrt{k} - 1}{k\sqrt{k}} \cdot \frac{k\sqrt{k}}{\sqrt{k}} = 1$ \rightarrow $\sum \frac{\sqrt{k} - 1}{k\sqrt{k}}$ \downarrow ∞

$\sum b_k$ \downarrow ∞ \parallel $b_k < a_k$ \rightarrow $\sum a_k$ \downarrow ∞
 \rightarrow $\# 702$

13. $\sum_{k=1}^{\infty} \frac{k^k}{(k+1)^{2k}} = \sum \left[\frac{k}{(k+1)^2} \right]^k$

but $\lim_{k \rightarrow \infty} \frac{k}{(k+1)^2} = 0$ $\# 9.1$ \rightarrow \sum \downarrow ∞

$\lim_{k \rightarrow \infty} \frac{k}{(k+1)^2} = 0$ $\# 9.1$ \rightarrow \sum \downarrow ∞

14. $\sum \frac{\cos(k\pi)}{k} = \sum \frac{(-1)^k}{k}$ \rightarrow $\sum \frac{1}{k}$ \downarrow ∞ $\# 11$
 $a_k = \frac{1}{k}$

$\lim_{k \rightarrow \infty} \frac{1}{k} = 0$

$k+1 > k$ \rightarrow $a_{k+1} = \frac{1}{k+1} < \frac{1}{k} = a_k$

but $\sum \frac{\cos(k\pi)}{k}$ \downarrow ∞

\rightarrow $\sum \cos(k\pi)$

စာလုံးတိုင်း၏ $\sum \left| \frac{\cos k\pi}{k} \right|$ သို့မဟုတ်

$$\sum \left| \frac{\cos k\pi}{k} \right| = \sum \frac{1}{k} \text{ သို့မဟုတ် } \sum \frac{1}{k} \text{ သို့မဟုတ် } \sum \frac{1}{k}$$

အားဖြင့် $\sum \frac{\cos k\pi}{k}$ သို့မဟုတ်

15. $\sum \frac{(-1)^k 4^{k-1}}{k 5^{k+1}}$ စာလုံးတိုင်း၏

သို့မဟုတ် $\sum \left| \frac{(-1)^k 4^{k-1}}{k 5^{k+1}} \right| = \sum \frac{4^{k-1}}{k 5^{k+1}} = \sum \frac{4^k}{20k 5^k}$ a_k

Ratio test

$$\frac{a_{k+1}}{a_k} = \frac{4^{k+1}}{20(k+1)5^{k+1}} \cdot \frac{20k 5^k}{4^k} = \frac{4}{20 \cdot 5} \cdot \frac{k}{k+1}$$

$$\lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} = \frac{4}{20 \cdot 5} = \frac{1}{25} < 1 \text{ သို့မဟုတ် } \sum \frac{4^k}{20k 5^k} \text{ သို့မဟုတ်}$$

အားဖြင့် $\sum \frac{(-1)^k 4^{k-1}}{k 5^{k+1}}$ သို့မဟုတ် စာလုံးတိုင်း၏ သို့မဟုတ်

16. $\sum_{k=1}^{\infty} \frac{1}{\ln(k+7)} (x-2)^k$ a_k

$$\left| \frac{a_{k+1}}{a_k} \right| = \left| \frac{(x-2)^{k+1}}{\ln(k+8)} \cdot \frac{\ln(k+7)}{(x-2)^k} \right| = \frac{\ln(k+7)}{\ln(k+8)} |x-2|$$

$$\lim_{k \rightarrow \infty} \frac{\ln(k+7)}{\ln(k+8)} |x-2| = 1 \cdot |x-2| < 1 \text{ သို့မဟုတ် } -1 < x-2 < 1$$

$$\lim_{y \rightarrow \infty} \frac{\ln(y+7)}{\ln(y+8)} = \lim_{y \rightarrow \infty} \frac{y+7}{y+8} = 1$$

$$-1 < x < 3$$

बसो $x = 1$

सुसमासु $\sum \frac{1}{\ln(k+7)} (-1)^k$ $a_k = \frac{1}{\ln(k+7)}$

$\lim_{k \rightarrow \infty} \frac{1}{\ln(k+7)} = 0$

$k+1+7 > k+7$

$\ln(k+8) > \ln(k+7)$

$a_{k+1} = \frac{1}{\ln(k+8)} < \frac{1}{\ln(k+7)} = a_k$

$a_{k+1} < a_k$
 याने? सुसमासु

अन्तिम $\sum \frac{(-1)^k}{\ln(k+7)}$ सुसमासु

बसो $x = 3$

$\sum \frac{1}{\ln(k+7)}$ a_k

$b_k = \frac{1}{k}$

$\sum \frac{1}{k}$ सुसमासु

बसुसमासु $n \neq 10$

$\lim_{k \rightarrow \infty} \frac{a_k}{b_k} = \lim_{k \rightarrow \infty} \frac{k}{\ln(k+7)} = \lim_{x \rightarrow \infty} \frac{1}{\frac{1}{x+7}} = \lim_{x \rightarrow \infty} x+7 = \infty$

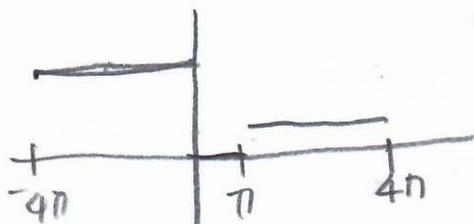
सुसमासु $\sum \frac{1}{\ln(k+7)}$ सुसमासु

अन्तिम सुसमासु $[1, 3)$ सुसमासु 1

17.1 $f(x) = 3x \sin x$
 $f(-x) = 3(-x) \sin(-x) = -3x(-\sin x) = 3x \sin x = f(x)$
 सुसमासु सुसमासु

17.2 $f(x) = x^2 + \cos x$
 $f(-x) = (-x)^2 + \cos(-x) = x^2 + \cos x = f(x)$ सुसमासु

18.



เขียนฟังก์ชันเป็น a_0, a_n, b_n

9

$$L = 4\pi$$

$$a_0 = \frac{1}{8\pi} \left\{ \int_{-4\pi}^0 3 dx + \int_{\pi}^{4\pi} 1 dx \right\} = \frac{1}{8\pi} [3(4\pi) + (3\pi)] = \frac{15\pi}{8\pi} = \frac{15}{8}$$

$$a_n = \frac{1}{4\pi} \left\{ 3 \int_{-4\pi}^0 \cos\left(\frac{n\pi x}{4\pi}\right) dx + \int_{\pi}^{4\pi} \cos\left(\frac{n\pi x}{4\pi}\right) dx \right\}$$

$$= \frac{1}{4\pi} \left\{ 3 \cdot \frac{4}{n} \left[\sin\left(\frac{nx}{4}\right) \right]_{-4\pi}^0 + \frac{4}{n} \left[\sin\left(\frac{nx}{4}\right) \right]_{\pi}^{4\pi} \right\}$$

$$= \frac{1}{4\pi} \left\{ \frac{12}{n} \left[\sin(0) - \sin(-n\pi) \right] + \frac{4}{n} \left[\sin(n\pi) - \sin\left(\frac{n\pi}{4}\right) \right] \right\}$$

$$= \frac{1}{4\pi} \cdot \frac{4}{n} (-\sin\left(\frac{n\pi}{4}\right)) = \frac{-\sin\left(\frac{n\pi}{4}\right)}{n\pi}$$

$$b_n = \frac{1}{4\pi} \left\{ 3 \int_{-4\pi}^0 \sin\left(\frac{nx}{4}\right) dx + \int_{\pi}^{4\pi} \sin\left(\frac{nx}{4}\right) dx \right\}$$

$$= \frac{1}{4\pi} \left\{ -3 \cdot \frac{4}{n} \left[\cos\left(\frac{nx}{4}\right) \right]_{-4\pi}^0 - \frac{4}{n} \left[\cos\left(\frac{nx}{4}\right) \right]_{\pi}^{4\pi} \right\}$$

$$= \frac{1}{4\pi} \left\{ \frac{-12}{n} \left[1 - \cos(n\pi) \right] - \frac{4}{n} \left[\cos(n\pi) - \cos\left(\frac{n\pi}{4}\right) \right] \right\}$$

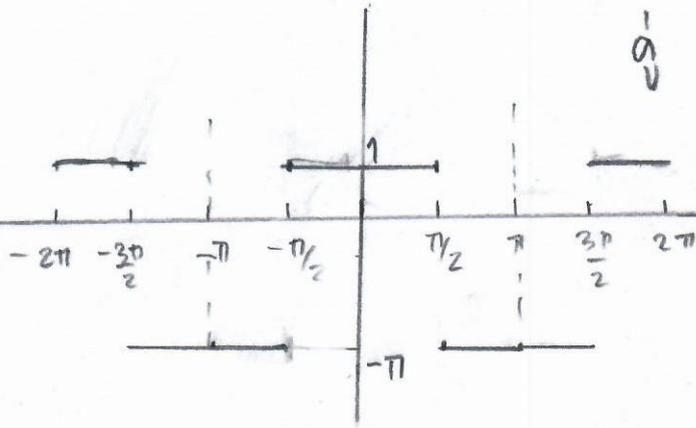
$$= \frac{1}{4\pi} \left\{ \frac{-12}{n} - \frac{8}{n} (-1)^n + \frac{4}{n} \cos\left(\frac{n\pi}{4}\right) \right\}$$

$$FS(f) = \frac{15}{8} + \sum_{n=1}^{\infty} \left(\frac{-\sin\left(\frac{n\pi}{4}\right)}{n\pi} \right) \cos\left(\frac{nx}{4}\right) + \sum_{n=1}^{\infty} \frac{1}{4\pi} \left(\frac{-12}{n} - \frac{8}{n} (-1)^n + \frac{4}{n} \cos\left(\frac{n\pi}{4}\right) \right) \sin\left(\frac{nx}{4}\right)$$

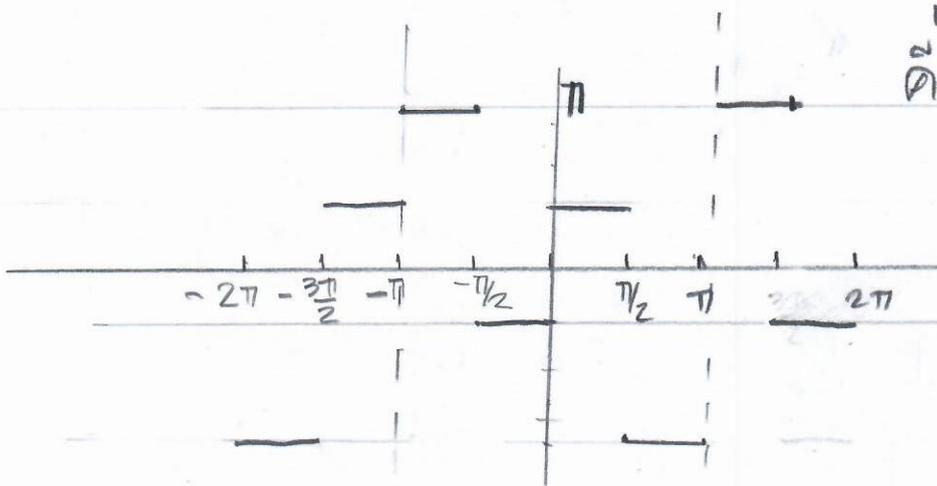
Emu

$$19. \quad f(x) = \begin{cases} 1, & 0 < x < \frac{\pi}{2} \\ -1, & \frac{\pi}{2} < x < \pi \end{cases}$$

19.1



19.2



Fourier cosine

परिधि $2L$

$$L = \pi$$

$$\frac{n\pi x}{L} = \frac{n\pi x}{\pi} = nx$$

$$a_0 = \frac{1}{\pi} \int_0^{\pi} f(x) dx = \frac{1}{\pi} \left\{ \int_0^{\pi/2} 1 dx + \int_{\pi/2}^{\pi} (-1) dx \right\} = \frac{1}{\pi} \left(\frac{\pi}{2} - \frac{\pi^2}{2} \right) = \frac{(1-\pi)}{2}$$

$$a_n = \frac{2}{\pi} \left\{ \int_0^{\pi/2} \cos(nx) dx - \int_{\pi/2}^{\pi} \cos(nx) dx \right\}$$

$$= \frac{2}{\pi} \left\{ \frac{1}{n} [\sin(nx)]_0^{\pi/2} - \frac{1}{n} [\sin(nx)]_{\pi/2}^{\pi} \right\}$$

$$= \frac{2}{n\pi} \left\{ (\sin(\frac{n\pi}{2}) - \sin 0) - (\sin(n\pi) - \sin(\frac{n\pi}{2})) \right\}$$

$$= \frac{2(1+\pi) \sin(\frac{n\pi}{2})}{n\pi}$$

Fourier cosine $\frac{1-\pi}{2} + \sum_{n=1}^{\infty} \frac{2(1+\pi) \sin(\frac{n\pi}{2}) \cos(nx)}{n\pi}$

Fourier sine

$$\begin{aligned}
b_n &= \frac{2}{\pi} \left\{ \int_0^{\pi/2} \sin(nx) dx - \pi \int_{\pi/2}^{\pi} \sin(nx) dx \right\} \\
&= \frac{2}{\pi} \left\{ -\frac{1}{n} [\cos(nx)]_0^{\pi/2} + \frac{\pi}{n} [\cos(nx)]_{\pi/2}^{\pi} \right\} \\
&= \frac{2}{n\pi} \left\{ -\cos\left(\frac{n\pi}{2}\right) + \cos 0 + \pi \left(\cos(n\pi) - \cos\left(\frac{n\pi}{2}\right) \right) \right\} \\
&= \frac{2}{n\pi} \left(-(1+\pi) \cos\left(\frac{n\pi}{2}\right) + 1 + \pi(-1)^n \right)
\end{aligned}$$

Fourier sine $\sum_{n=1}^{\infty} \frac{2}{n\pi} \left(-(1+\pi) \cos\left(\frac{n\pi}{2}\right) + 1 + \pi(-1)^n \right) \cos(nx)$