206171 REVIEW EXERCISE (MIDTERM 1/62)

1. Find all values of c such that the following system has exactly one solution.

$$\begin{aligned} 9x - 3y &= 12 \\ cx + y &= 4 \end{aligned}$$

2. Find all values of m and n such that the following system has many solutions.

$$\begin{aligned} 4x + my &= 10 \\ nx - 6y &= -20 \end{aligned}$$

3. Joe and Joy bought two kilograms of mangoes and three kilograms of apples. It cost 400 baht. They then bought another kilogram of mangoes and two more kilograms of apples for 200 baht. **Write a system of linear equations** to find the cost of a kilogram of mangoes and a kilogram of apples. **Do not solve the problem.**

4. Solve the following system using Gauss-Jordan elimination.

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 1 \\ 2x_1 - 3x_2 - 2x_3 &= 3 \\ x_1 - x_2 - 5x_3 &= 3 \end{aligned}$$

5. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 0 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ using row operations.

6. Solve the following matrix equation **using matrix inverse method**.

$$\begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 25 \\ 14 \end{bmatrix} = \begin{bmatrix} 13 \\ 22 \end{bmatrix}$$

7. A company manufactures sedans and minivans. Each sedan and minivan must pass through 2 departments: assembly and finishing. The relevant manufacturing data are given in the table.

	Assembly department	Finishing department
Labor-hours per sedan	1	2
Labor-hours per minivan	1	1
Maximum labor-hours available per week	18	25

The company decides that the number of minivans produced should be at least half the number of sedans produced.

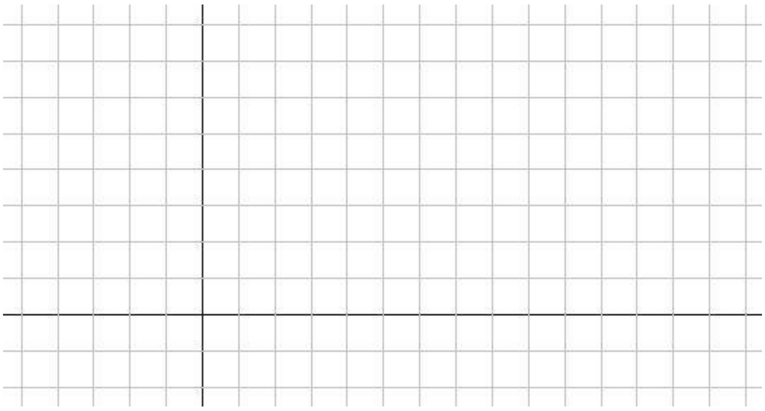
Write a linear programming problem to find the number of sedans and minivans that should be manufactured per week in order to maximize the profit provided that the profits on a sedan and a minivan are \$5,000 and \$2,000, respectively.

8. Solve the following linear programming problem using graphs.

Minimize $Z = 4x + 2y$

Subject to

$$3x + y \geq 8$$
$$3x - 2y \leq 2$$
$$y \leq 5$$
$$x, y \geq 0$$



9. Solve the following LP problem using simplex method.

Maximize $z = 3x + y$

Subject to

$$-x + y \leq 1$$
$$x - y \leq 1$$
$$y \leq 2$$
$$x, y \geq 0$$

The optimal solution is ____________________________The optimal value is___________________

10. The following tableau was obtained from solving a linear program with nonnegative variables y_1, y_2 and two inequalities. The objective function is maximized and slack variables s_1, s_2 were added.

	y_1	y_2	s_1	s_2	
	2 -2	1 2	1 0	0 1	20 80
	-5	-7	0	0	0
	2 -6	1 0	1 -2	0 1	20 40
	9	0	7	0	140

Write the corresponding LP problem.

11. Given the **original problem**,

Minimize $C = 4x_1 + 2x_2 + x_3$

Subject to

$$x_1 - x_2 + 2x_3 \geq 8$$
$$x_1 + x_2 - x_3 \leq 2$$
$$2x_1 - 3x_2 + 4x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0.$$

a) Write the **dual problem** of the original problem.

b) This is the final tableau in the dual of the maximization problem.

	y_1	y_2	y_3	*	*	*	
S_1	0	-3/2	0	1	0	-1/2	7/2
Y_3	0	-1/2	1	0	1	1/2	5/2
Y_1	1	-1/2	0	0	2	3/2	11/2
Z	0	1/2	0	0	11	19/2	63/2

The optimal solution is ___________________________The optimal value is___________________

12. Consider an LP problem. Find the modified problem for Big M method. Write the preliminary simplex tableau and then the initial simplex tableau and **circle the first pivot**.

Maximize $P = 2x_1 - x_2$

Subject to

$$x_1 + x_2 \le 5$$
$$5x_1 + 3x_2 \ge 30$$
$$x_1, x_2 \ge 0$$

The modified problem is:

Preliminary simplex tableau		
Initial simplex tableau (Make the values of the artificial variable columns in the last row equal to zero and circle the first pivot point)		

13. Starbugs Company fries bugs (ants, bees and crickets) to sell at their two shops. Everyday ant farm (A), bee farm (B), and cricket farm (C) provide freshly caught ants, bees and crickets to the two Starbugs' shops. Maximum quantity of bugs caught per day and the transportation cost are given in the table below.

Farm	**Max Quantity (kg)**	**Cost (baht/kg)**
Ant farm (A)	100	50
Bee farm (B)	5	1000
Cricket farm (C)	25	200

If each shop is required to sell at least 50 kilograms of fried bugs per day, write an LP problem to minimize the cost of transporting bugs from the farms to the shops. Do not solve the problem.

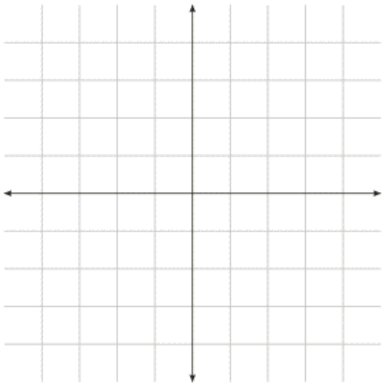
14. Suppose a maximum problem can be written to the initial simplex tableau as below.

	x	y	z	S_1	S_2	S_3	a_3	a_4	
S_1	-1	-2	1	1	0	0	0	0	16
S_2	4	-1	-6	0	1	0	0	0	3
a_3	1	3	5	0	0	-1	1	0	18
a_4	4	2	0	0	0	0	0	1	15
P	-5M+1	-5M	-5M+2	0	0	M	0	0	-44M

Complete the next pivoting step.

15. Sketch the graph of each function below. Find its domain and range.

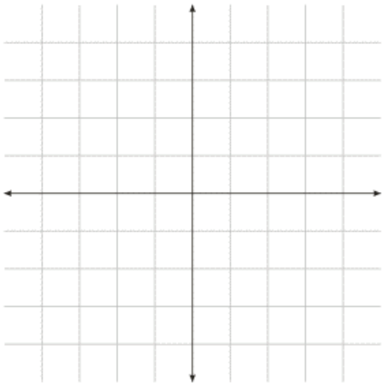
a) $f(x) = \sqrt{x-1} + 3$



Domain of f is ____________________________

Range of f is ____________________________

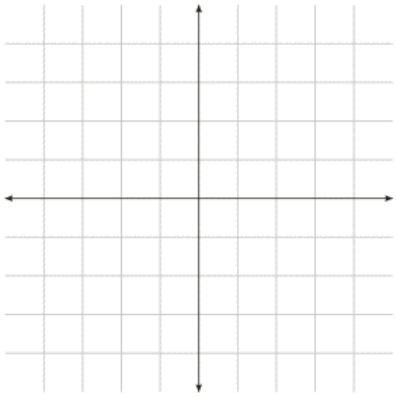
b) $g(x) = \dfrac{x^2 + x}{x}$



Domain of g is ____________________________

Range of g is ____________________________

c) $h(x) = \log_2(4x)$



Domain of h is ___________________________

Range of h is ___________________________

18. Let $f : \mathbb{R} \to \mathbb{R}$ be a function given by $f(x) = \begin{cases} \left|2 - \dfrac{3}{x}\right| & ; \ x > 0 \\ -\sin(\pi x^2) & ; \ x \le 0 \end{cases}$.

Find $f(2) - f\left(-\dfrac{1}{\sqrt{6}}\right)$.

19. Give an example for each of the following.

Function	**Example**
A quadratic function	
A polynomial function with degree 5	
An exponential function with base 0.5	
A logarithmic function with base 3	
A rational function with a horizontal asymptote $y = 0$	
A rational function with a vertical asymptote $x = 1$	

20. Give all the asymptotes to the given functions.

	Horizontal Asymptote(s)	**Vertical Asymptote(s)**
$\dfrac{15x^3+3x}{5-5x^3}$		
$\dfrac{x+2}{x^2+2x-8}$		

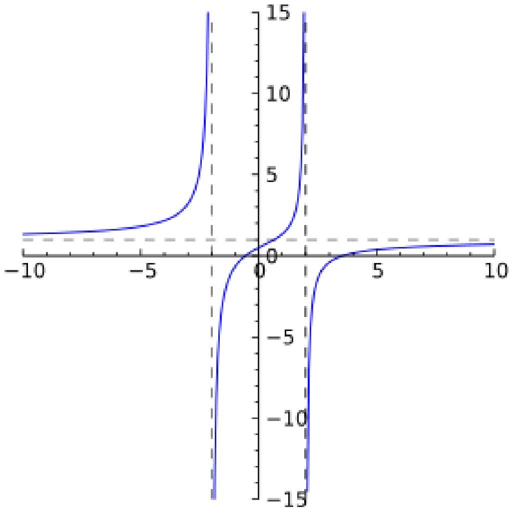
21. Consider the graph of a function f below.

a) Does f have horizontal asymptote(s)?

If yes, the horizontal asymptote(s) of f is (are)

b) Does f have vertical asymptote(s)?

If yes, the vertical asymptote(s) of f is (are)



22. Answer the following.

a) If $\log_b 2 = 10$, then $b^{20} =$ ______________________

b) If $\ln 2 = M$, then $\left(e^M\right)^3 =$ ______________________

c) If $\log_{10} N = -3$, then $N =$ ______________________

d) If $f(x) = 5x + \ln x$, then $f(1) =$ ______________________

e) $e^{\ln(7.5)} =$ __________________

f) If $\sin(x) = \cos(0)$, $0 \le x \le 2\pi$, then $x =$ ____________________

23. Solve the following equations.

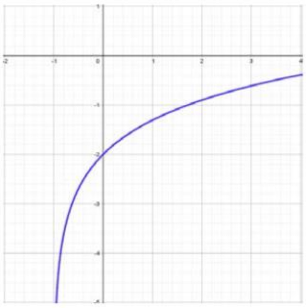
a) $\log_2(x+3)+\log_2(2-x)=2$

b) $\sqrt{2}\cdot 4^{\left(x-\frac{1}{2}\right)}=8\sin\left(\frac{\pi}{2}\right)$

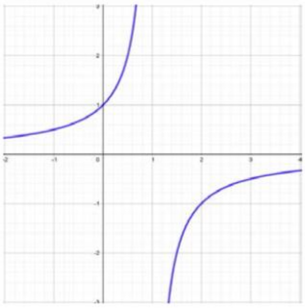
24. Match each function with its graph.

22.1	$y=-\left(\frac{1}{4}\right)^{x-1}+1$	________________
22.2	$y=\ln(x+1)-2$	________________
22.3	$y=\frac{1}{x-1}$	________________
22.4	$y=\frac{1}{4}e^{x}$	________________

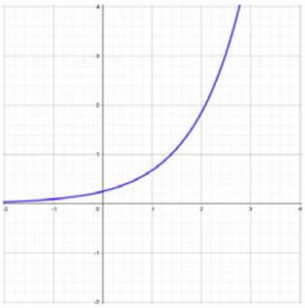
(A)



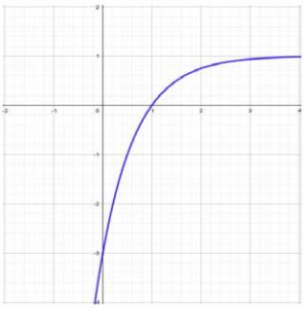
(B)



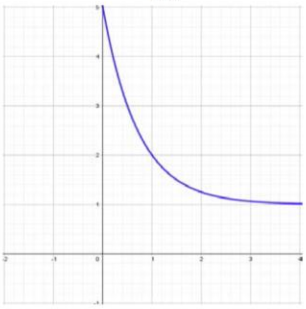
(C)



(D)



(E)



(F)