การบ้านชุดที่ 7 แบบฝึก 2.2.2

ชุด7 น.1/12

3) $I_1 = \int \arctan x \, dx$

ให้ $u = \arctan x \Rightarrow du = \dfrac{1}{1+x^2} dx$

$dv = dx \Rightarrow \int dv = \int dx \Rightarrow v = x$

สูตร by part

$$\int u \, dv = u \cdot v - \int v \, du$$

$$\int \arctan x \, dx = (\arctan x)(x) - \int x \left(\frac{1}{1+x^2} \right) dx$$

$$= x \arctan x - \int \frac{x}{1+x^2} dx$$

Consider $I_2 = \int \dfrac{x}{1+x^2} dx$

ให้ $U = 1 + x^2$

$\dfrac{du}{dx} = 2x \Rightarrow dx = \dfrac{du}{2x}$

$$\therefore I_2 = \int \frac{x}{u} \left(\frac{du}{2x} \right)$$

$$= \frac{1}{2} \int \frac{du}{u}$$

$$= \frac{1}{2} \ln u + c$$

$$= \frac{1}{2} \ln |1+x^2| + c$$

$$\therefore I_1 = \int \arctan x \, dx = x \arctan x - \frac{1}{2} \ln |1+x^2| + c \quad \#$$

ปูอาจ 8.2/12

5) $I = \int x \operatorname{cosec}^2 x \, dx$

ให้ $u = x \Rightarrow du = dx$

$dv = \operatorname{cosec}^2 x \, dx \Rightarrow \int dv = \int \operatorname{cosec}^2 x \, dx$

$$v = -\cot x$$

จากสูตร by part

$$\int u \, dv = u \cdot v - \int v \, du$$

$$\therefore I = \int x \operatorname{cosec}^2 x \, dx = -x \cot x + \int \cot x \, dx$$

$$= -x \cot x + \ln|\sin x| + C$$

11) $I = \int \frac{xe^x}{(x+1)^2} dx$

ให้ $u = xe^x \Rightarrow du = (xe^x + e^x)dx$

$$du = e^x(x+1)dx$$

และ $\int dv = \int \frac{1}{(x+1)^2} dx \Rightarrow v = -\frac{1}{x+1}$

จาก สูตร by part

$$I = \int u \, dv = u \cdot v - \int v \, du$$

$$\int \frac{xe^x}{(x+1)^2} dx = -\frac{xe^x}{(x+1)} + \int \frac{1}{(x+1)} (e^x(x+1))dx$$

$$= -\frac{xe^x}{(x+1)} + \int e^x dx$$

$$= -\frac{xe^x}{x+1} + e^x + C \quad \#$$

7) $I = \int e^{-x}\cos x\,dx$

ให้ $u = e^{-x} \Rightarrow du = -e^{-x}dx$

และ $dv = \cos x\,dx \Rightarrow \int dv = \int \cos x\,dx$

$v = \sin x$

สูตร by part

$\therefore\ I = \int u\,dv = u\cdot v - \int v\,du$

$\int e^{-x}\cos x\,dx = e^{-x}\sin x + \int e^{-x}\sin x\,dx \quad -\ (1)$

consider $\int e^{-x}\sin x\,dx$

ให้ $u = e^{-x} \Rightarrow du = -e^{-x}dx$

และ $dv = \sin x\,dx \Rightarrow \int dv = \int \sin x\,dx$

$v = -\cos x$

จาก $\int u\,dv = u\cdot v - \int v\,du$

$\int e^{-x}\sin x\,dx = -e^{-x}\cos x - \int (\cos x)e^{-x}dx \quad -\ (2)$

นำ (2) แทนลง (1) ;

$\int e^{-x}\cos x\,dx = e^{-x}\sin x + \left[-e^{-x}\cos x - \int e^{-x}\cos x\,dx\right]$

$2\int e^{-x}\cos x\,dx = e^{-x}\sin x - e^{-x}\cos x$

$\int e^{-x}\cos x\,dx = \dfrac{e^{-x}\sin x - e^{-x}\cos x}{2} + c$ #

ปุดา P.4/12

(9) $I = \int (\ln x)^2 \, dx$

ให้ $u = (\ln x)^2$

และ $dv = dx \;=\; \int dv = \int dx$

$\therefore v = x$

จากสูตร by part

$$\int u \, dv = uv - \int v \, du$$

$$\therefore I = \int (\ln x)^2 dx = (\ln x)^2 \cdot x - \int x \, d(\ln x)^2$$

$$= x(\ln x)^2 - \int \cancel{x} \cdot \frac{2\ln x}{\cancel{x}} \, dx$$

$$= x(\ln x)^2 - 2\int \ln x \, dx$$

$$= x(\ln x)^2 - 2\left[x\ln x - \int x \, d(\ln x)\right] \text{ by part}$$

$$= x(\ln x)^2 - 2x\ln x + 2\int \cancel{x} \cdot \frac{1}{\cancel{x}} \, dx$$

$$= x(\ln x)^2 - 2x\ln x + 2x + C$$

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ex. 2.2.3

1) $\int \cos^3 x \sqrt{\sin x}\, dx$

$= \int \cos^2 x \cos x \sqrt{\sin x}\, dx$

$= \int (1-\sin^2 x) \cos x \sqrt{\sin x}\, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x \Rightarrow dx = \frac{du}{\cos x}$

$= \int (1-u^2) \cancel{\cos x} (u^{\frac{1}{2}}) \left(\frac{du}{\cancel{\cos x}}\right)$

$= \int u^{\frac{1}{2}} - u^{\frac{3}{2}}\, du$

$= \frac{2}{3} u^{\frac{3}{2}} - \frac{2}{5} u^{\frac{5}{2}} + C$

$= \frac{2}{3} (\sin x)^{\frac{3}{2}} - \frac{2}{5} (\sin x)^{\frac{5}{2}} + c$ ##

3) $\int \cos^2 \pi x \, dx$

$= \int \frac{1}{2}\left[1 + \cos 2\pi x\right] dx$

$= \frac{1}{2}\int \left(1 + \cos 2\pi x\right) dx$

$= \frac{1}{2}\left[x + \frac{\sin 2\pi x}{2\pi}\right] + c$ #

5) $\int \sin^3 x \cos^4 x \, dx$

$= \int \sin^2 x \sin x \cos^4 x \, dx$

$= \int (1-\cos^2 x) \sin x \cos^4 x \, dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x \Rightarrow dx = \frac{du}{-\sin x}$

$= \int (1-u^2) \cancel{\sin x} (u^4) \left(\frac{du}{\cancel{-\sin x}}\right)$

$= \int u^6 - u^4 \, du$

$= \frac{u^7}{7} - \frac{u^4}{4} + C$

$= \frac{\cos^7 x}{7} - \frac{\cos^4 x}{4} + C$

7) $\int \sin\theta \sin 3\theta \, d\theta$

$$\int \frac{1}{2}\left[\cos(\theta - 3\theta) - \cos(\theta + 3\theta)\right] d\theta$$

$$= \int \frac{1}{2}\left[\cos(-2\theta) - \cos(4\theta)\right] d\theta$$

$$= \frac{1}{2}\left(\frac{\sin(-2\theta)}{-2} - \left(-\frac{\sin 4\theta}{4}\right)\right) + C$$

$$= -\frac{\sin 2\theta}{4} + \frac{\sin 4\theta}{8} + C \quad \#$$

9) $\int \cos 4\theta \cdot \cos(-3\theta) \, d\theta$

$$= \int \frac{1}{2}\left[\cos(4\theta + (-3\theta)) + \cos(4\theta - (-3\theta))\right] d\theta$$

$$= \int \frac{1}{2}\left[\cos(\theta) + \cos(7\theta)\right] d\theta$$

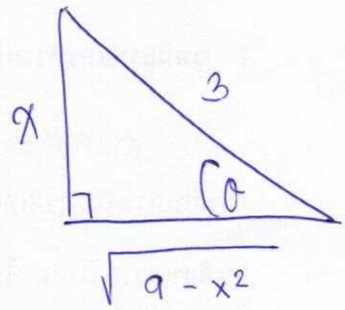
$$= \frac{1}{2}\left(\frac{\sin\theta}{1} + \frac{\sin 7\theta}{7}\right) + C$$

$$= \frac{\sin\theta}{2} + \frac{\sin 7\theta}{14} + C \quad \#$$

แบบฝึก 2.2.4 ชุดที่ 7

1) $\displaystyle\int \frac{1}{x^2\sqrt{9-x^2}}\,dx$

$x = 3\sin\theta \Rightarrow \sin\theta = \frac{x}{3} \Rightarrow$



$dx = 3\cos\theta\,d\theta$

$$= \int \frac{1}{(3\sin\theta)^2\sqrt{9-(3\sin\theta)^2}}\,(3\cos\theta\,d\theta)$$

$$= \int \frac{3\cos\theta\,d\theta}{(9\sin^2\theta)(\sqrt{9(1-\sin^2\theta)})}$$

$$= \int \frac{\cancel{3\cos\theta}}{9\sin^2\theta\,\cancel{3\cos\theta}}\,d\theta$$

$$= \frac{1}{9}\int \operatorname{cosec}^2\theta\,d\theta$$

$$= -\frac{1}{9}\cot\theta + c$$

$$= -\frac{1}{9}\left(\frac{\sqrt{9-x^2}}{x}\right) + c$$

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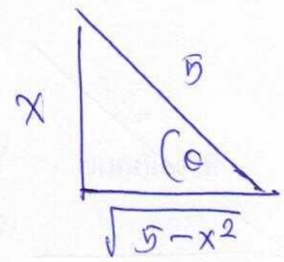
3) $\displaystyle\int \frac{1}{(25-x^2)^{\frac{3}{2}}}\,dx$

$x = 5\sin\theta \Rightarrow \sin\theta = \frac{x}{5} \Rightarrow$



$dx = 5\cos\theta\, d\theta$

$$= \int \frac{5\cos\theta\, d\theta}{(25-(5\sin\theta)^2)^{\frac{3}{2}}}$$

$$= \int \frac{5\cos\theta}{(25(1-\sin^2\theta))^{\frac{3}{2}}}\, d\theta$$

$$= \int \frac{5\cos\theta}{(5\cos\theta)^3}\, d\theta$$

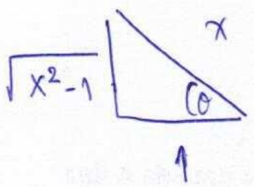
$$= \frac{1}{25}\int \sec^2\theta\, d\theta$$

$$= \frac{1}{25}\tan\theta + C$$

$$= \frac{1}{25}\left(\frac{x}{\sqrt{5-x^2}}\right) + C \quad //$$

⑤ $\int \frac{x^2}{(x^2-1)^{5/2}} dx$

$x = \sec\theta \Rightarrow$



$dx = \sec\theta\tan\theta$

$= \int \frac{\sec^2\theta\ (\sec\theta\tan\theta\, d\theta)}{(\sec^2\theta - 1)^{\frac{5}{2}}}$

$= \int \frac{\sec^3\theta\tan\theta\, d\theta}{\tan^5\theta}$

$= \int \frac{\sec^3\theta}{\tan^4\theta} d\theta$

$= \int \frac{1}{\cos^3\theta}\left(\frac{\cos^4\theta}{\sin^4\theta}\right) d\theta$

$= \int \frac{\cos\theta}{\sin^4\theta} d\theta$

$u = \sin\theta$

$\frac{du}{d\theta} = \cos\theta$

$d\theta = \frac{du}{\cos\theta}$

$= \int \frac{\cos\theta}{u^4}\left(\frac{du}{\cos\theta}\right)$

$= \int u^{-4} du$

$= \frac{u^{-3}}{-3} + C$

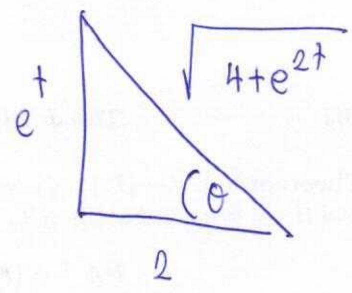
$= -\frac{1}{3u^3} + C$

$= -\frac{1}{3\sin^3\theta} + C$

$= -\frac{1}{3}\left(\frac{x}{\sqrt{x^2-1}}\right)^3 + C$

7) $\int \frac{e^t}{(4+e^{2t})^{3/2}}\,dt$

$e^t = 2\tan\theta \Rightarrow \tan\theta = \frac{e^t}{2} \Rightarrow$



$e^t\,dt = 2\sec^2\theta\,d\theta$

$dt = \frac{2\sec^2\theta\,d\theta}{e^t}$

$= \int \frac{\cancel{e^t}}{(4+(2\tan\theta)^2)^{\frac{3}{2}}}\left(\frac{2\sec^2\theta\,d\theta}{\cancel{e^t}}\right)$

$= \int \frac{2\sec^2\theta}{(4(1+\tan^2\theta))^{\frac{3}{2}}}\,d\theta$

$= \int \frac{2\sec^2\theta}{(2\sec\theta)^3}\,d\theta$

$= \frac{1}{4}\int \frac{1}{\sec\theta}\,d\theta$

$= \frac{1}{4}\int \cos\theta\,d\theta$

$= \frac{1}{4}\sin\theta + c$

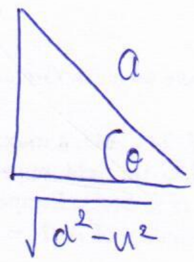
$= \frac{1}{4}\left(\frac{e^t}{\sqrt{4+e^{2t}}}\right) + c$ ⊗

ปอ7 8.12/12

9.) $\int \sqrt{a^2 - u^2}\, du$

$u = a\sin\theta \quad \Rightarrow \quad \sin\theta = \frac{u}{a} \quad \Rightarrow$

$du = a\cos\theta\, d\theta$



$= \int \sqrt{a^2 - a^2\sin^2\theta}\,(a\cos\theta\, d\theta)$

$= \int \sqrt{a^2(1-\sin^2\theta)}\,(a\cos\theta\, d\theta)$

$= \int (a\cos\theta)(a\cos\theta)\, d\theta$

$= a^2 \int \cos^2\theta\, d\theta$

$= a^2 \int \frac{1}{2}\left[1 + \cos 2\theta\right] d\theta$

$= a^2 \left[\frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]\right] + C$

$= \frac{1}{2}a^2\theta + \frac{a^2\sin 2\theta}{4} + C$

$= \frac{1}{2}a^2 \arcsin\left(\frac{u}{a}\right) + \frac{a^2\sin\theta\cos\theta}{2} + C$

$= \frac{1}{2}a^2 \arcsin\left(\frac{u}{a}\right) + \frac{a^2}{2}\left(\frac{u}{a}\right)\left(\frac{\sqrt{a^2-u^2}}{a}\right) + C$

$= \frac{1}{2}a^2 \arcsin\left(\frac{u}{a}\right) + \frac{u\sqrt{a^2-u^2}}{2a^2} + C$ #