

# บทเรียน การอินทิเกรต ชุด 6 แบบฝึก 2.1

1)  $\int (8y - \frac{2}{\sqrt[3]{y}}) dy$   
 $= \int (8y - 2y^{\frac{1}{3}}) dy$   
 $= \frac{8y^2}{2} - \frac{2y^{\frac{4}{3}}}{\frac{4}{3}} + C$   
 $= 4y^2 - \frac{3}{2}y^{\frac{4}{3}} + C \quad \#$

2)  $\int \frac{\sqrt{t} + \sqrt{t}}{t^2} dt$   
 $= \int \frac{t^{\frac{1}{2}} + t^{\frac{1}{2}}}{t^2} dt$   
 $= \int (t^{-\frac{1}{2}} + t^{-\frac{3}{2}}) dt$   
 $= 2t^{\frac{1}{2}} - 2t^{-\frac{1}{2}} + C \quad \#$

5)  $\int \cos 2x - \operatorname{cosec}^2 x dx$   
 $= \frac{\sin 2x}{2} + \cot x + C$

6)  $\int (e^{3x} - 3e^{-2x}) dx$   
 $= \frac{e^{3x}}{3} + \frac{3}{2}e^{-2x} + C$

9)  $\int \frac{1}{3+x^2} dx$   
 $= \int \frac{1}{(\sqrt{3})^2 + x^2} dx$   
 $= \frac{1}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right) + C \quad \#$

906 2.2.1

2)  $I = \int \frac{2^{-\frac{1}{x^2}} dx}{x^3}$

Let  $u = -\frac{1}{x^2} = -x^{-2}$

$\frac{du}{dx} = +2x^{-3} = \frac{2}{x^3} \Rightarrow dx = \frac{x^3}{2} du$

$\therefore I = \int \frac{2^u}{x^3} \left( \frac{x^3}{2} du \right)$

$= \frac{1}{2} \int 2^u du$

$= \frac{1}{2} \left( \frac{2^u}{\ln 2} \right) + C$

$= \frac{2^{-\frac{1}{x^2}}}{2 \ln 2} + C \quad \#$

6)  $I = \int \frac{e^{\sqrt{x}} dx}{\sqrt{x}}$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow dx = 2\sqrt{x} du$

$I = \int \frac{e^u}{\sqrt{x}} (2\sqrt{x} du)$

$= 2 \int e^u du$

$= 2e^u + C = 2e^{\sqrt{x}} + C \quad \#$

$$4) I = \int \frac{y}{\sqrt{2y^2+1}} dy$$

$$\text{für } u = 2y^2 + 1$$

$$\frac{du}{dy} = 4y \Rightarrow dy = \frac{du}{4y}$$

$$I = \int \frac{y}{\sqrt{u}} \left( \frac{du}{4y} \right)$$

$$= \frac{1}{4} \int \frac{1}{\sqrt{u}} du$$

$$= \frac{1}{4} \int u^{-\frac{1}{2}} du$$

$$= \frac{1}{2} u^{\frac{1}{2}} + C$$

$$= \frac{1}{2} (2y^2 + 1) + C \quad \#$$

$$10) I = \int e^y \sin(e^y) dy$$

$$\text{für } u = e^y$$

$$\frac{du}{dy} = e^y \Rightarrow dy = \frac{du}{e^y}$$

$$\rightarrow I = \int \cancel{e^y} \sin(u) \left( \frac{du}{\cancel{e^y}} \right)$$

$$= \int \sin(u) du$$

$$= -\cos u + C$$

$$= -\cos(e^y) + C \quad \#$$

$$14) I = \int \frac{\ln^2 x}{x} dx$$

$$\text{Qu } u = \ln x$$

$$\frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x du$$

$$\therefore I = \int \frac{u^2}{\cancel{x}} (\cancel{x} du)$$

$$= \int u^2 du$$

$$= \frac{u^3}{3} + C$$

$$= \frac{\ln^3 x}{3} + C \quad \#$$

$$16) \int x \sqrt{x-3} dx$$

$$\text{Qu } u = x-3 \rightarrow x = u+3 \quad |$$

$$\frac{du}{dx} = 1 \Rightarrow dx = du \quad |$$

$$\int (u+3)(\sqrt{u}) du$$

$$= \int (u\sqrt{u} + 3\sqrt{u}) du$$

$$= \int (u^{\frac{3}{2}} + 3u^{\frac{1}{2}}) du$$

$$= \frac{2}{5} u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + C$$

$$= \frac{2}{5} (x-3)^{\frac{5}{2}} + 2(x-3)^{\frac{3}{2}} + C \quad \#$$

17)

ကိစ္စအရ အကွေ့အကောက်ပေးပေးရန် အားပေးပေးရန်  $a(t) = (2t+3)^{-3}$  မြေပေါ်/အောက်  $2$  ကိစ္စအရ အကွေ့အကောက်ပေးပေးရန်  $t=0$  မှ  $4$  မြေပေါ်/အောက် အကွေ့အကောက်ပေးပေးရန်  $2$  ကိစ္စအရ

အဖြေ အကွေ့အကောက်ပေး  $a(t) = (2t+3)^{-3}$  မြေပေါ်/အောက်  $2$  ကိစ္စအရ  $v(0) = 4$  မြေပေါ်/အောက် အကွေ့အကောက်ပေး  $v(2)$

$$v(t) = \int a(t) dt = \int (2t+3)^{-3} dt$$

$$= \frac{1}{2} \int (2t+3)^{-3} d(2t+3)$$

$$= \frac{1}{2} \frac{(2t+3)^{-2}}{-2} + C = -\frac{1}{4} (2t+3)^{-2} + C$$

အကွေ့အကောက်ပေး  $v(0) = 4$

$\therefore -\frac{1}{4 \cdot 9} + C = 4 \Rightarrow C = 4 + \frac{1}{36}$

$\therefore v(t) = -\frac{1}{4(2t+3)^2} + 4 + \frac{1}{36}$

$\therefore v(2) = \frac{1}{4(7)^2} + 4 + \frac{1}{36}$

$$= \frac{4}{140} + \frac{1}{4} \left( \frac{1}{7^2} + \frac{1}{36} \right)$$

$$= 4 + \frac{1}{2} \cdot \frac{58}{36} = 4 + \frac{19}{882} = 4 \frac{19}{882} \#$$

(18) နံပါတ် ၁၂၀ တွင် နေရာပေးထားသော  $y = f(x)$  ကို  $8x + \operatorname{cosec}^2 x$  ဖြစ်အောင်  
 ရှာဖွေပါ။  $(\frac{\pi}{2}, -7)$  ကို နေရာပေးထားသော နေရာပေးပါ။

ဖြေဆို နေရာပေးထားသော  $y' = f'(x) = 8x + \operatorname{cosec}^2 x$   
 နေရာ  $y = f(x)$  ကို  $(x, y) = (\frac{\pi}{2}, -7)$

စာ  $y' = f'(x) = 8x + \operatorname{cosec}^2 x$

$$\therefore y = f(x) = \int (8x + \operatorname{cosec}^2 x) dx$$

$$= 4x^2 - \cot x + C$$

ဆိုပါစကားကို  $(x, y) = (\frac{\pi}{2}, -7)$  ကို နေရာပေးပါ။

$$\therefore -7 = 4\left(\frac{\pi}{2}\right)^2 - \cot\left(\frac{\pi}{2}\right) + C$$

$$-7 = \pi^2 - 0 + C$$

$$\therefore C = -7 - \pi^2$$

$$\therefore f(x) = 4x^2 - \cot x - 7 - \pi^2$$

#

(19) နံပါတ် ၁၂၀ တွင် နေရာပေးထားသော  $\frac{dN}{dt} = 0.25N$  ကို  $N = 200$  နေရာပေးပါ။  $N$  ကို  $t = 8$  နေရာပေးပါ။

ဖြေဆို နေရာပေးထားသော  $N(0) = 200$  နေရာပေးပါ။  $N(t)$

စာ  $\frac{dN}{dt} = 0.25N$

$$\therefore \frac{1}{N} dN = \frac{0.25}{1} dt \Rightarrow \int \frac{1}{N} dN = \int \frac{0.25}{1} dt$$

$$\frac{N^2}{2} + C = \frac{N^2}{8} + C$$

$$\int \frac{1}{N} dN = \int \frac{1}{4} dt$$

$$\therefore \ln N = \frac{t}{4} + C \Rightarrow N = e^{\frac{t}{4} + C} = A e^{\frac{t}{4}}$$

$N(t) = 200 e^{\frac{t}{4}}$

စာ  $N(0) = 200 \Rightarrow 200 = A e^{\frac{0}{4}} \Rightarrow A = 200$

$$N(8) = 200 e^{\frac{8}{4}} = 200 e^2$$

$$\therefore N(8) = 200 e^2$$

$$\therefore N(8) = 200 e^{\frac{8}{4}} = 200 e^2$$

၂၇၆၈.၆၁၆ #